Selfish routing

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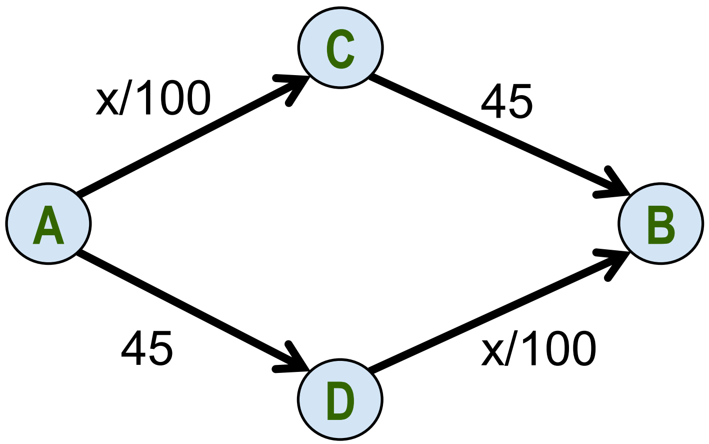
Slides based on Social and Economic Networks: Models and Applications, lecture 11, by Borodin and Boutilier

A Traffic Network

Suppose 4000 drivers must get from A to B each morning, travel time depends on the traffic. If *x* cars on a link (segment) travel time is as labeled, varies on A—C and D—B but is fixed on A—D and C—B



E.g. if 3000 take route A-C-B,
1000 take A-D-B
then route C: 75 mins
route D: 55 mins

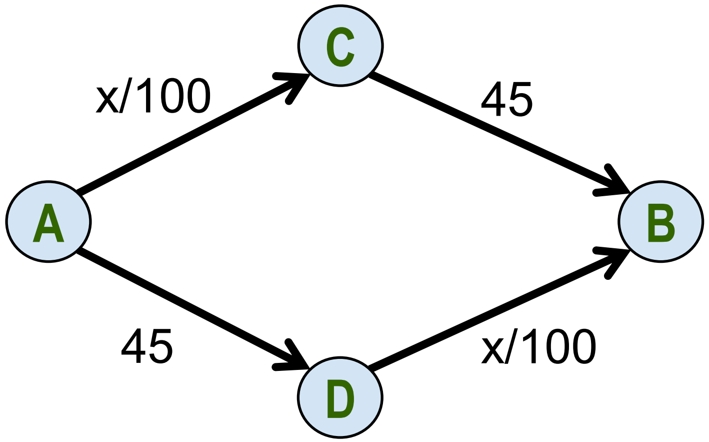
Traffic Flow in Equilibrium

Suppose 4000 cars travel from A to B each morning

- *What is equilibrium traffic flow?*

Model as a game with 4000 players

- each driver can choose route A—C—B or A—D—B
- each driver prefers to minimise her *personal* travel time



Many Nash equilibria!

But all are “equivalent”:
- 2000 drivers take C
- 2000 drivers take D
- all travel times: 65mins

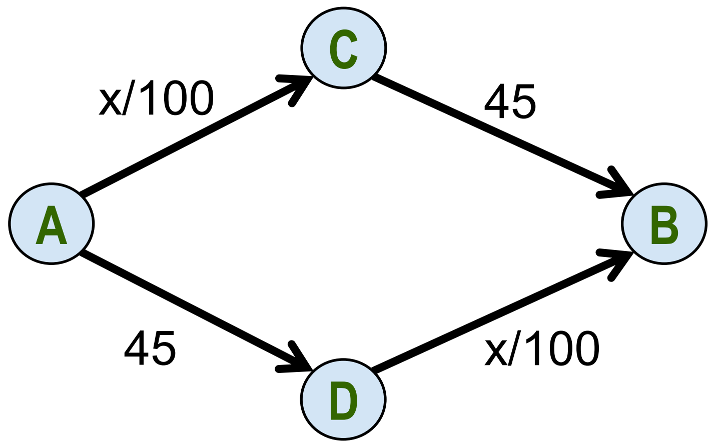
Traffic Flow in Equilibrium

(Any) profile <2000 C, 2000 D> is a NE

- each route is equally fast: 65 mins, no incentive to switch
- in fact, if a driver switches (e.g., from C to D): her travel time goes up from 65 mins to 65.01 mins
- How many NE? Combinations of 2000 out of 4000 $\approx 1.6 \times 10^{1202}$

Why is <n C, 4000-n D> not a NE if $n \neq 2000$?

- Any driver on slower route will want to switch to faster route



Social Optimality

"The" Nash equilibrium is in fact *socially optimal* and it is the only socially optimal way to arrange traffic

- it minimizes the population's total (equiv. average) commute time
- in the NE (2000/2000): everyone has 65 minute commute time
- if you shift balance to (2001 C, 1999 D):
  - *1999* drivers see commute time drop by 0.01 (64.99 mins)
  - but *2001* see commute time rise by 0.01 (65.01 mins)
  - total commute time goes up by 0.02 mins
- (2100 C, 1900D): total increase of 200 mins
- (3000 C, 1000D): total increase of 20,000 mins (about 2 weeks)
- (4000 C, 0D): total increase of 80,000 mins (almost 2 months)

How might NE emerge in practice?

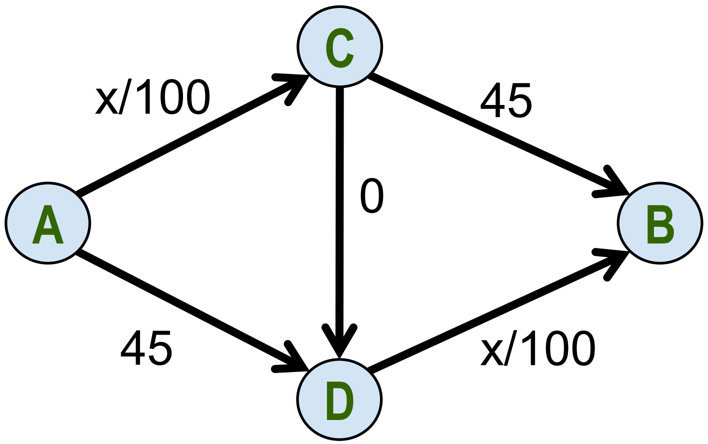
With 10^{1200} NE: ultimate equilibrium selection problem!

- 4000 drivers didn't call each other up this AM and coordinate
- Iterative process? Try out a route... if it's fast you stick, if it's slow you switch?
  - Suppose 4000 start with C: what do they do the next day?
  - All switch to D! Then all switch back to C, ...
- More likely, a probabilistic process, some people more amenable to switching than others... and the slower it is the more likely you are to switch... over time after a process of gradual adjustment leads to something that is approximate NE (e.g. RL with a low learning rate)

What's nice about this: self-organization based on self- interest makes everyone better off, indeed as well-off as possible, since it maximizes social welfare

Braess' Paradox

What happens to traffic patterns if we add a new superhighway to reduce everyone's commute time, link with much smaller time (zero)



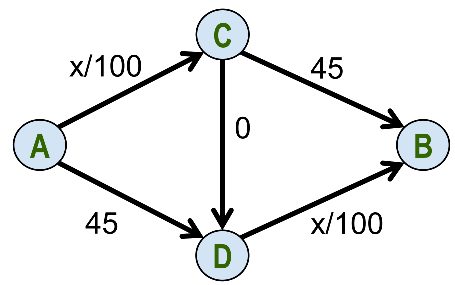
Braess' Paradox

Unique NE in new game:

- *everyone* takes A-C-D-E; commute increases from 65 to 80 mins!

Why?

- the *longest* links A-C and D-B can take is 40 mins (all 4000 on them)
- so A-C-D always faster than A-D; and D-B always faster than C-B
- so A-C-D-B is *dominant* for every driver



"Paradox":
adding capacity slowed everyone down

- named for discoverer (Diettrich Braess, 1968)
- *observed in real traffic situations*

Why does it happen?

Before new link:

- all routes from A-B required one 45 min link
- facilitates traffic split, easing congestion on A-C and D-B

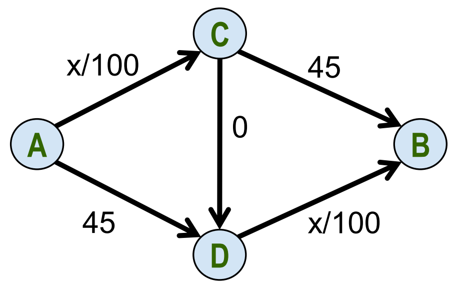
After new link:

- everyone can avoid 45 min link
- but *only one way to do so*: all traffic through C-D
- leaves both 45 min links (A-D, C-B) unused!

Is the new link really usefull?

Compare new link with ***imposed*** social optimum to no link

- Without new link: everyone takes 65 mins
- With new link, social optimum average commute time is 64.69 mins
  - 500 have A-C-D-B (45 mins)
  - 3500 have ADB or ACB (67.5 mins)
  - Total time saved: 1250 mins (avg. 19 sec per driver)
- Not Pareto improving: 500 people save a lot at expense of 3500 others



New link, if you can't impose social optimum:

- Average increases from 64.69 to 80 mins
- So allowing people to act in their own interests (equilibrium) causes a society (and in this case, every member of society) to suffer (aka Tragedy of the Commons)

Price of Anarchy

How much societal benefit do you sacrifice by letting everyone choose their own actions?

Or what is the ratio SCNE / OPT?

socially optimal profile (OPT)

social cost of the NE (SCNE)

This is bounded by 4/3 (for linear cost function),

In general, it can be exponential!

=> coordination mechanisms are necessary.

References

Visit Tim Toughgarden’s webpage http://theory.stanford.edu/~tim/

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