Neural Networks





Plan

- Perceptron
 - Linear discriminant
- Associative memories
 - Hopfield networks
 - Chaotic networks
- Multilayer perceptron
 - Backpropagation

- Historically, the first neural net
- Inspired by human brain
- Proposed
 - By Rosenblatt
 - Between 1957 et 1961
- The brain was appearing as the best computer
- Goal: associated input patterns to recognition outputs
- Akin to a linear discriminant















- Need an associated learning
 - Learning is supervised
 - Based on a couple input pattern and desired output
 - If the activation of output neuron is OK => nothing happens
 - Otherwise inspired by neurophysiological data
 - If it is activated : decrease the value of the connection
 - If it is unactivated : increase the value of the connection
 - Iterated until the output neurons reach the desired value

- Supervised learning
 - How to decrease or increase the connections ?
 - Learning rule of Widrow-Hoff
 - Closed to Hebbian learning

$$\mathbf{W}_{i,j}^{(t+1)} = \mathbf{W}_{i,j}^{(t)} + \mathbf{n}(t_j - \mathbf{O}_j) \mathbf{x}_i = \mathbf{W}_{i,j}^{(t)} + \Delta \mathbf{W}_{i,j}$$

$$\downarrow$$
Desired value of output neuron
Learning rate

Theory of linear discriminant

Compute: $g(x) = W^{T}x + W_{o}$ And:

Choose:

class 1 if g(x) > 0 class 2 otherwise

But how to find W on the basis of the data ?



Gradient descent:

$$\Delta W_i = -\eta \, \frac{\partial E}{\partial W_i}, \forall i$$

In general a sigmoid is used for the statistical interpretation: (0,1)

$$Y = 1/1 + \exp\left[-g(x)\right]$$

Easy to derive = Y(1-Y)

Class 1 if Y > 0.5 and 2 otherwise

The error could be least square: $(Y - Y_d)^2$

Or maximum likelihood: $-\sum Y_d \log Y + (1 - Y_d) \log(1 - Y)$

But at the end, you got the learning rule: $\Delta W = \eta \sum (Yd - Y)X_i$

Perceptron limitations

- Limitations
 - Not always easy to learn
 - But above all, cannot separate not linearly separable data

1,1

1.0

0.0

- Why so ?
 - The XOR kills NN researches

 for 20 years
 (Minsky and Papert were responsable)
- Consequence
 - We had to wait for the magical hidden layer
 - And for backpropagation

Associative memories

- Around 1970
- Two types
 Hetero-associative
 And auto-associative
- We will treat here only auto-associative
- Make an interesting connections between neurosciences and physics of complex systems
- John Hopfield

Auto-associative memories



Associative memories





It is again an hebbian learning rule

Associative memories



Hopfield

The newtork becomes a dynamical machine

It has been shown to converge into a fixed point

This fixed point is a minimal of a Lyapunov energy

These fixed point are used for storing «patterns » Discrete time and asynchronous updating input in {-1,1} $x_i \rightarrow sign(\Sigma_j w_{ij}x_j)$

Mémoires associatives



Hopfield

The learning is done by Hebbian learning



Over all patterns to learn:

$$\Delta W_{ij} = \sum_{patterns} X_i^p X_j^p$$

My researches: Chaotic encoding of memories in brain



Multilayer perceptron



Multilayer Perceptron



Error backpropagation

- Learning algorithm
- How it proceeds :
 - Inject an input
 - Get the output
 - Compute the error with respect to the desired output
 - Propagate this error back from the output layer to the input layer of the network
 - Just a consequence of the chaining derivative of the gradient descent

Select a derivable transfert function
 Classicaly used : The logistics

$$f(x) = \frac{1}{1 + e^{-x}}$$

And its derivative

$$f'(x) = f(x)[1 - f(x)]$$

The algorithm



1. Inject an entry

Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o



Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o
- 4. Compute the error output

5. Adjust Z on the basis of the error

Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
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- 4. Compute the error output

5. Adjust Z on the basis of the error

6. Compute the error on the hidden layer

Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o
- 4. Compute the error output

5. Adjust Z on the basis of the error

6. Compute the error on the hidden layer

7. Adjust W on the basis of this error

Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o
- 4. Compute the error output

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Algorithm



- →1. Inject an entry
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Neural network



<u>Simple linear discriminant</u>



Neural networks



Few layers - Little learning



Neural networks



More layers - More learning







suggestion de décision

Neural networks



<u>Tricks</u>

Favour simple NN (you can add the structure in

the error) Few layers are enough (theoretically only one)

Exploit cross validation...

