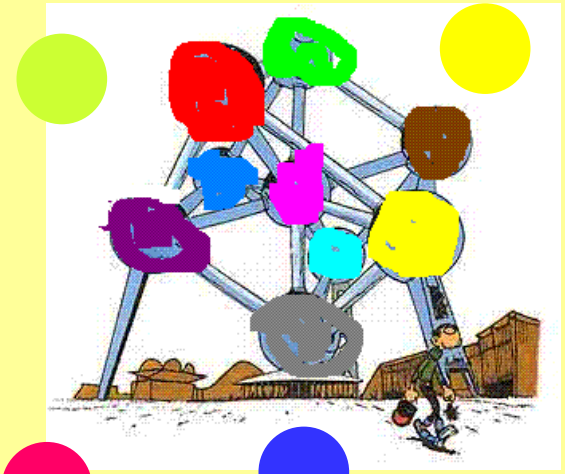
Networks and sciences: The story of the “small-world”

Hugues Bersini
IRIDIA – ULB

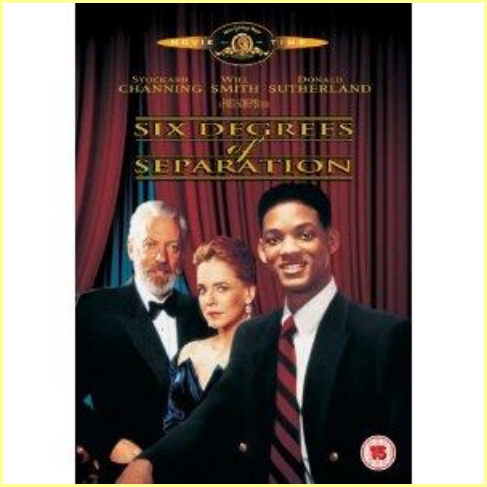


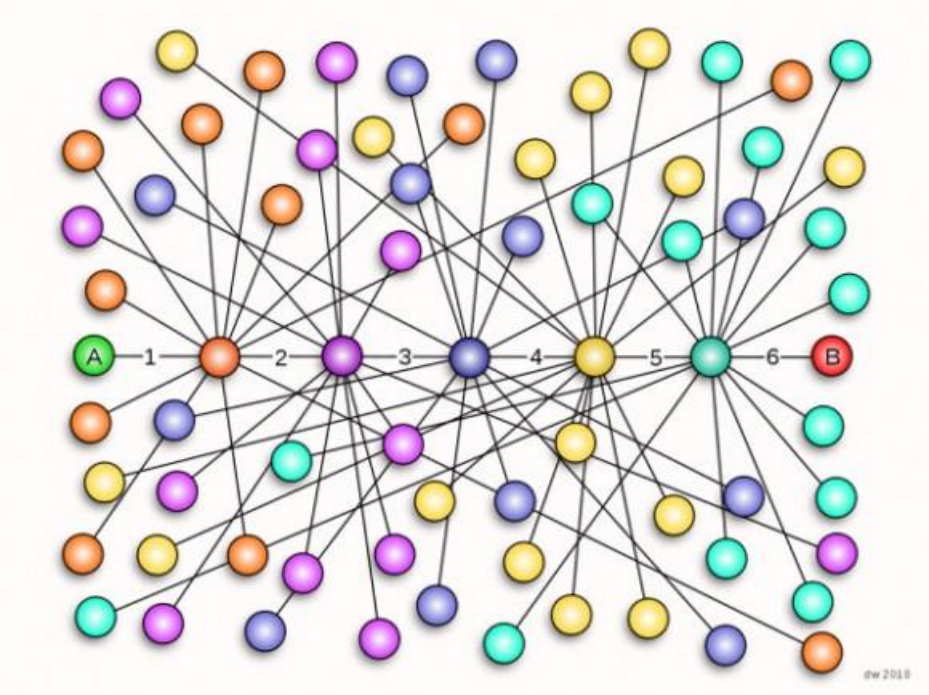
The story begins with Stanley Milgram (1933-1984)

In 1960, the famous experience of the submission to authority

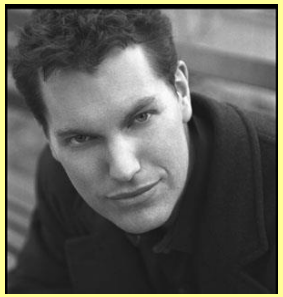
En 1967, the as famous experience of the six degrees of separation

Stanley Milgram




A
1
2
3
4
5
6
B

The story moves on with Watts and Strogatz (1998)



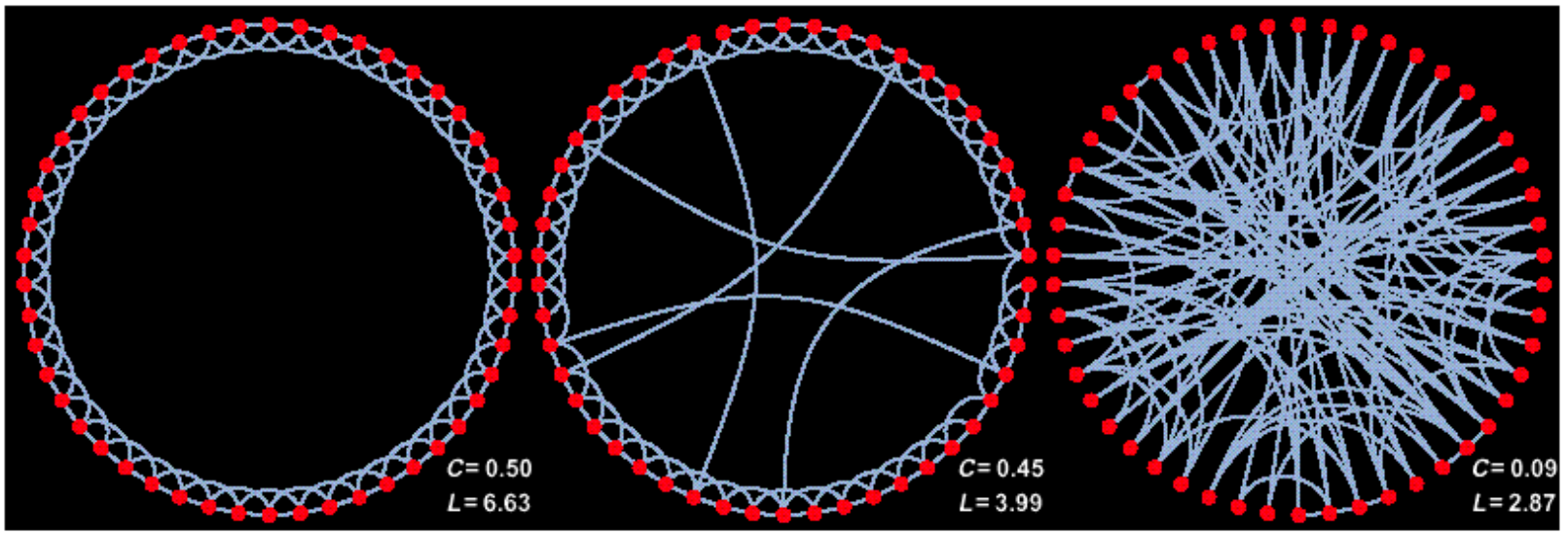
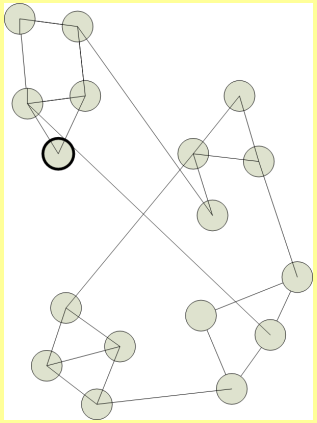
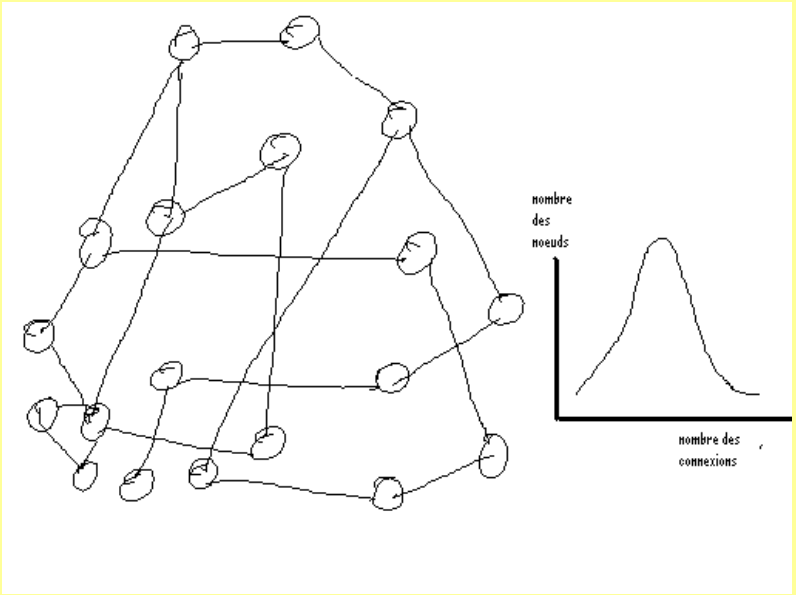


Figure 1. Watts-Strogatz model interpolates between a regular lattice *(left)* and a random graph *(right)*. Randomly rewiring just a few edges *(center)* reduces the average distance between nodes, L, but has little effect on the clustering coefficient, C. The result is a "small-world" graph.

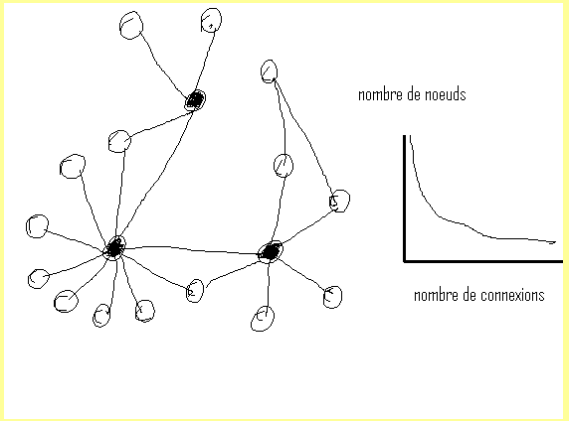
But networks are far from homogeneous!!



With aggregates

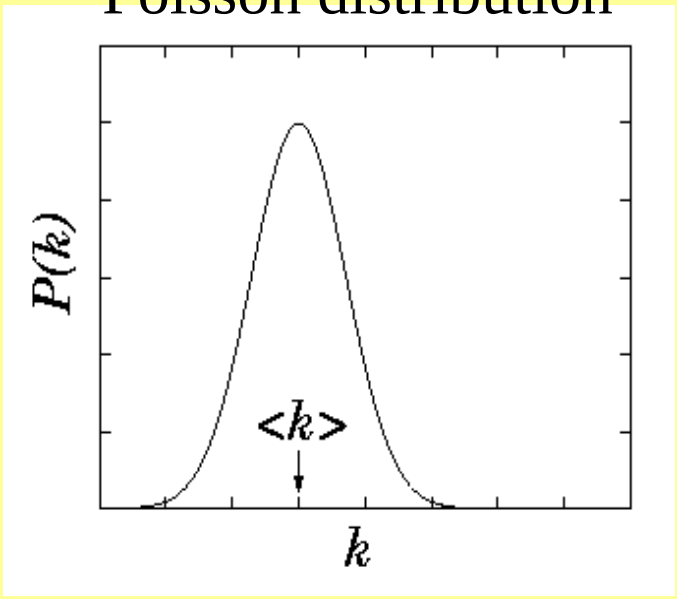


Random

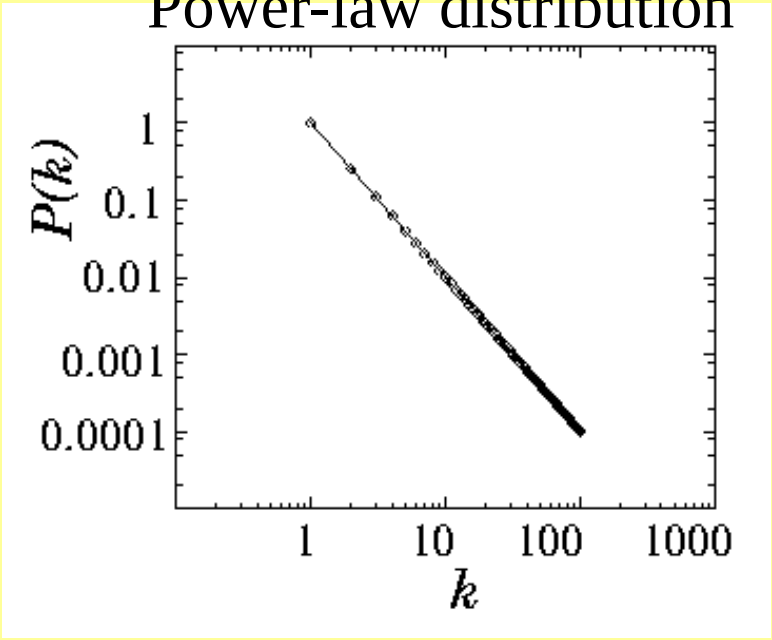


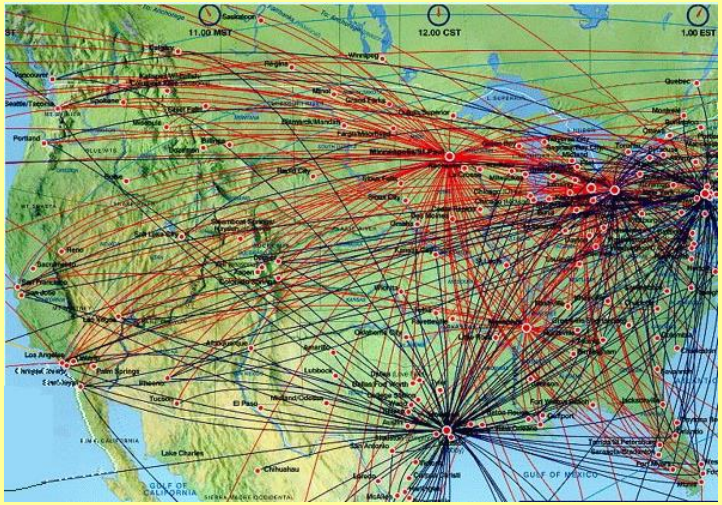
Scale-Free

Poisson distribution



Power-law distribution

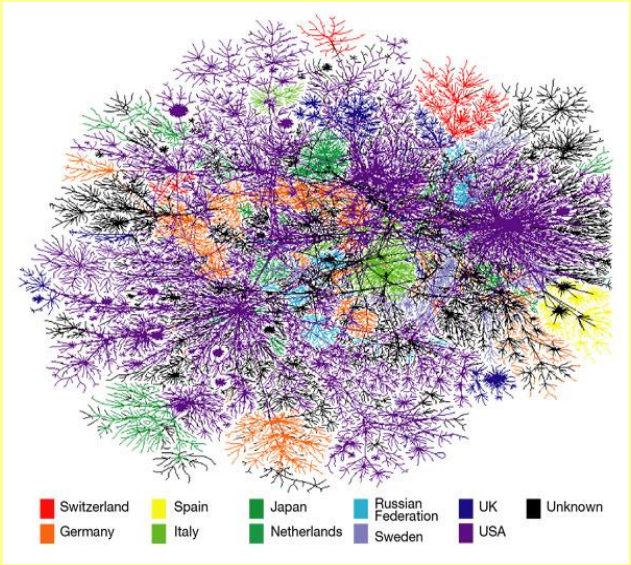


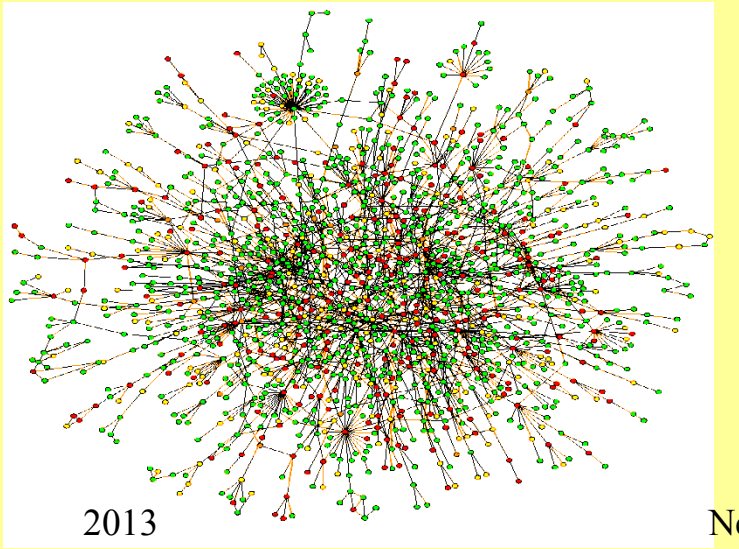


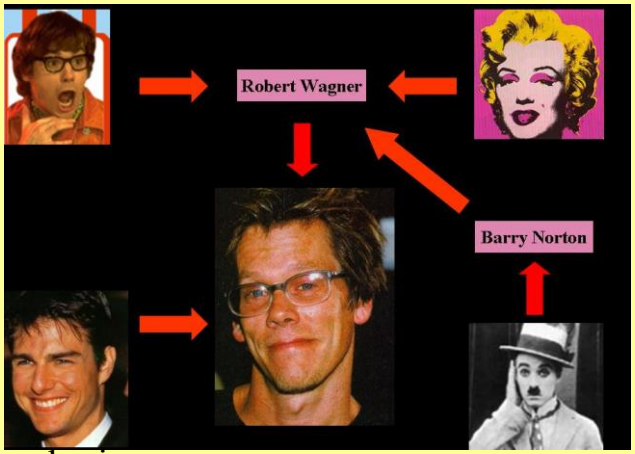
Exponential Network

Scale-free Network

The story ends up with Babarasi (in the years 2000)

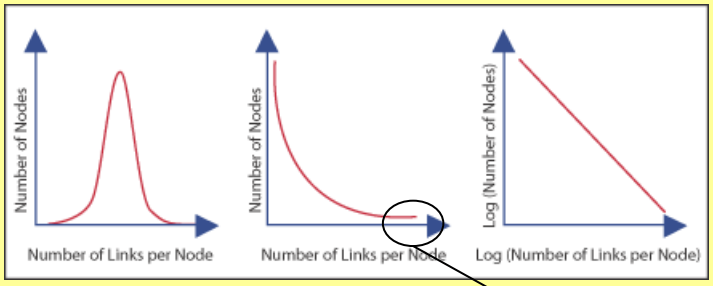






Networks are « scale-free »:

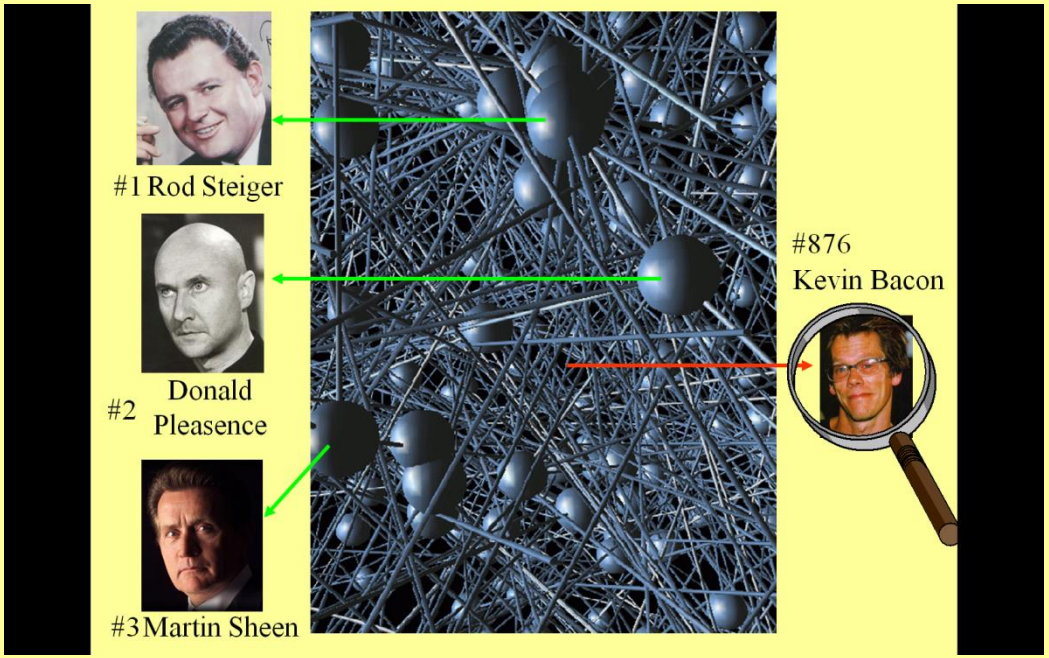
$$P(k) \sim k^{-\gamma}$$

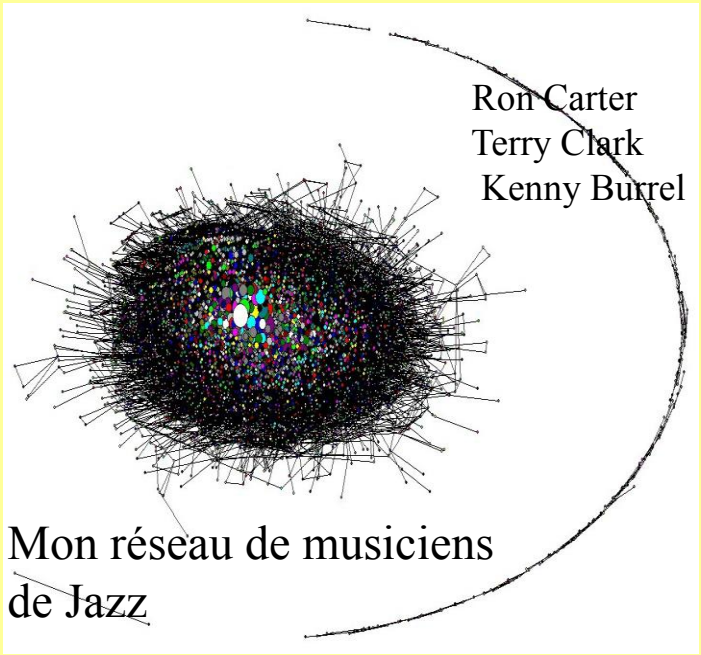


connecteurs

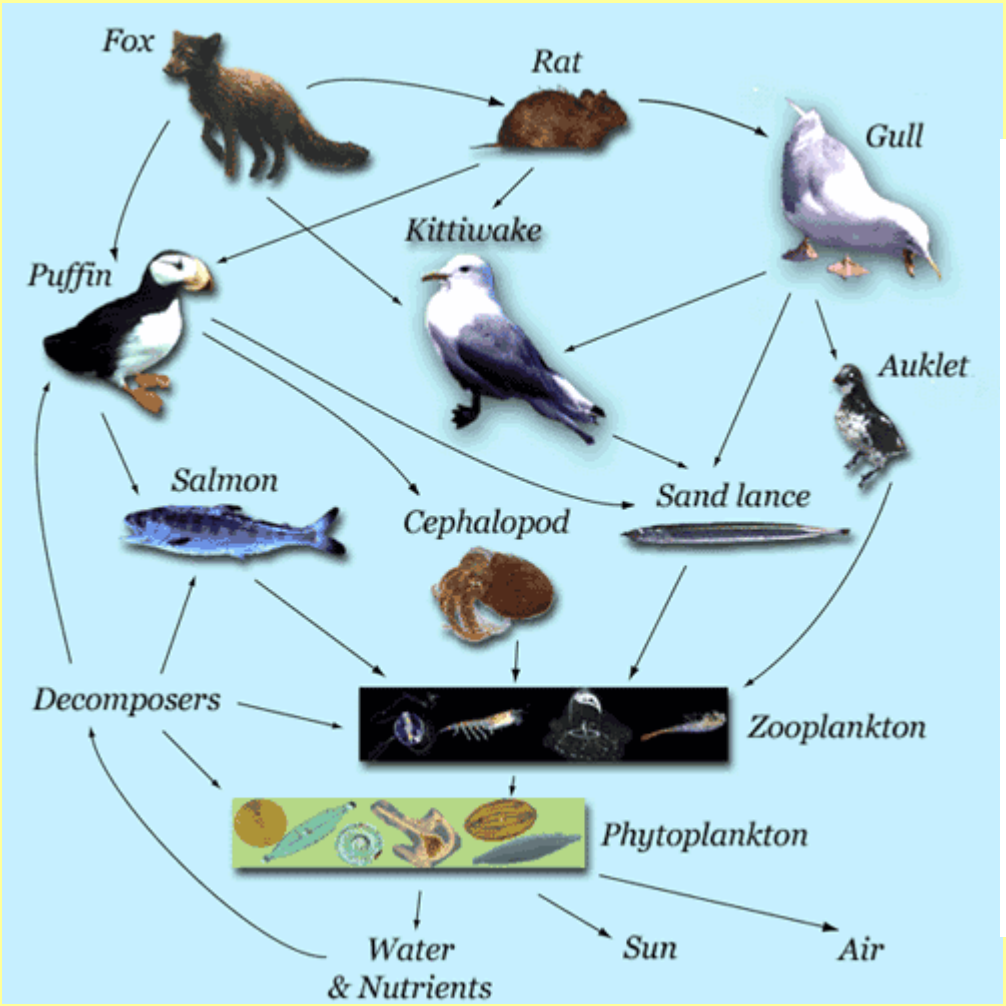
WWW (in)	Internet	Actor	Citation index	Sex Web	Cellular network	Phone call network	linguistics
$\gamma = 2.1$	$\gamma = 2.5$	$\gamma = 2.3$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 2.1$	$\gamma = 2.1$	$\gamma = 2.8$

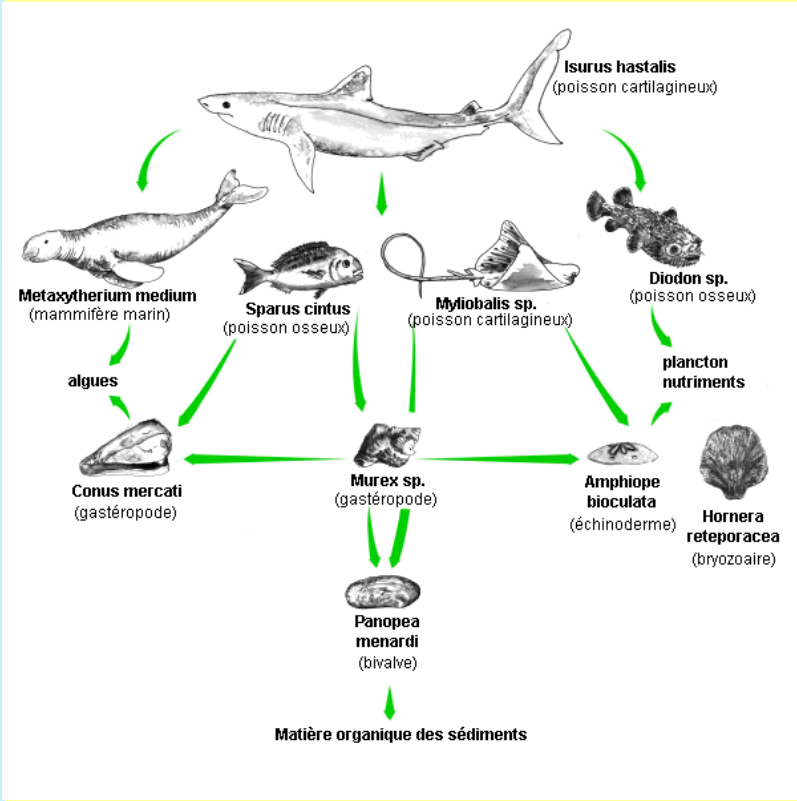
The kingdom of hubs





Trophic Network

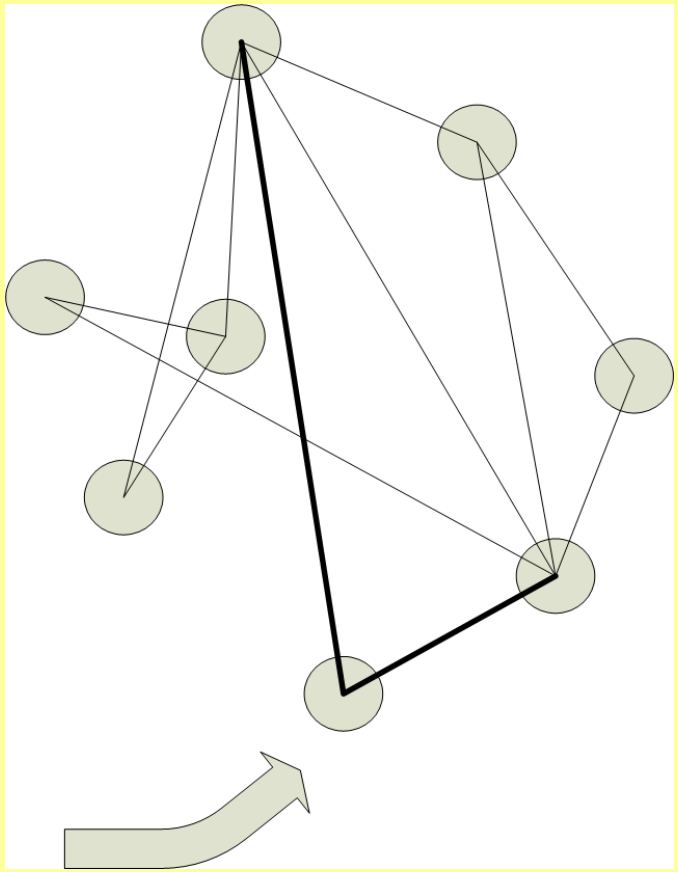




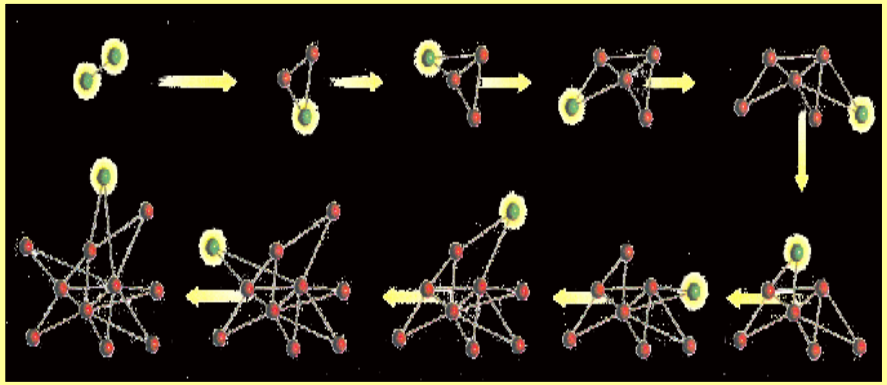
How topology and hubs influence the global behaviour of networks

- The small-world effect → very small distance among nodes
- Epidemic propagation (epidemiology or viral marketting)
- Study of robustness: target (on hubs) or random attacks

Why scale-free ?
The rule of preferential attachment



Rich get richer

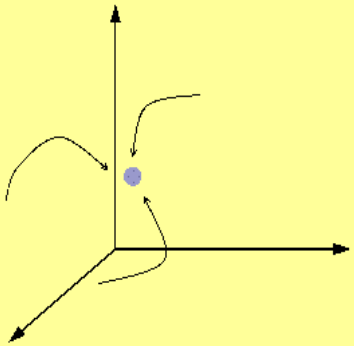


Research at IRIDIA on networks

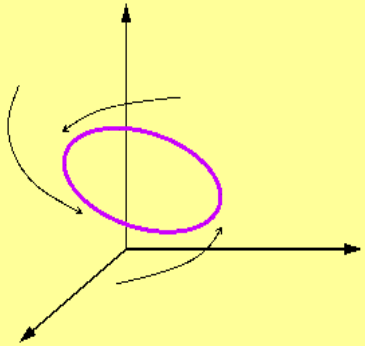
Network Dynamics

- Homogeneous units a_i (t) (the same temporal evolution – the same differential equations)
  - $da_i/dt = F(a_j, W_{ij}, I)$
- A given topology in the connectivity matrix: W_{ij}
- Entries I which perturb the dynamics and to which the network gives meaning (→ attractors)
- A very large family of concerned biological networks
  - Idiotypic immune network
  - Hopfield network
  - Coupled Map Lattice
  - Boolean network
  - Ecological network (Lokta-Volterra)
  - Genetic network

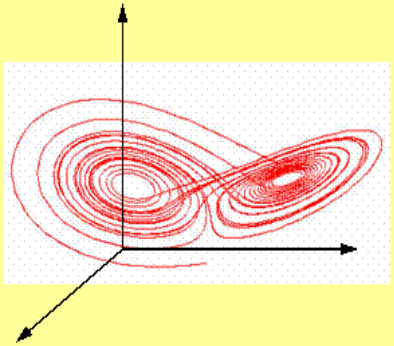
Dynamics



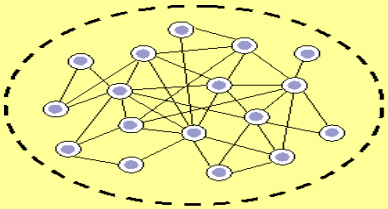
Attracteur point fixe



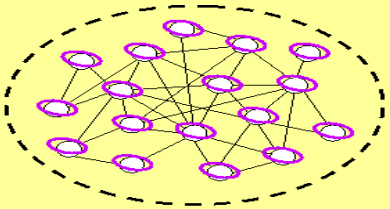
Attracteur cyclique



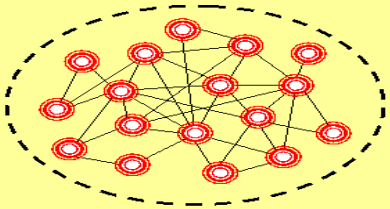
Attracteur chaotique



Réseau point fixe



Réseau cyclique



Réseau chaotique

Chemistry

Ecosystem: prey/predator

Brain, heart

Topology influences the dynamics of immune network

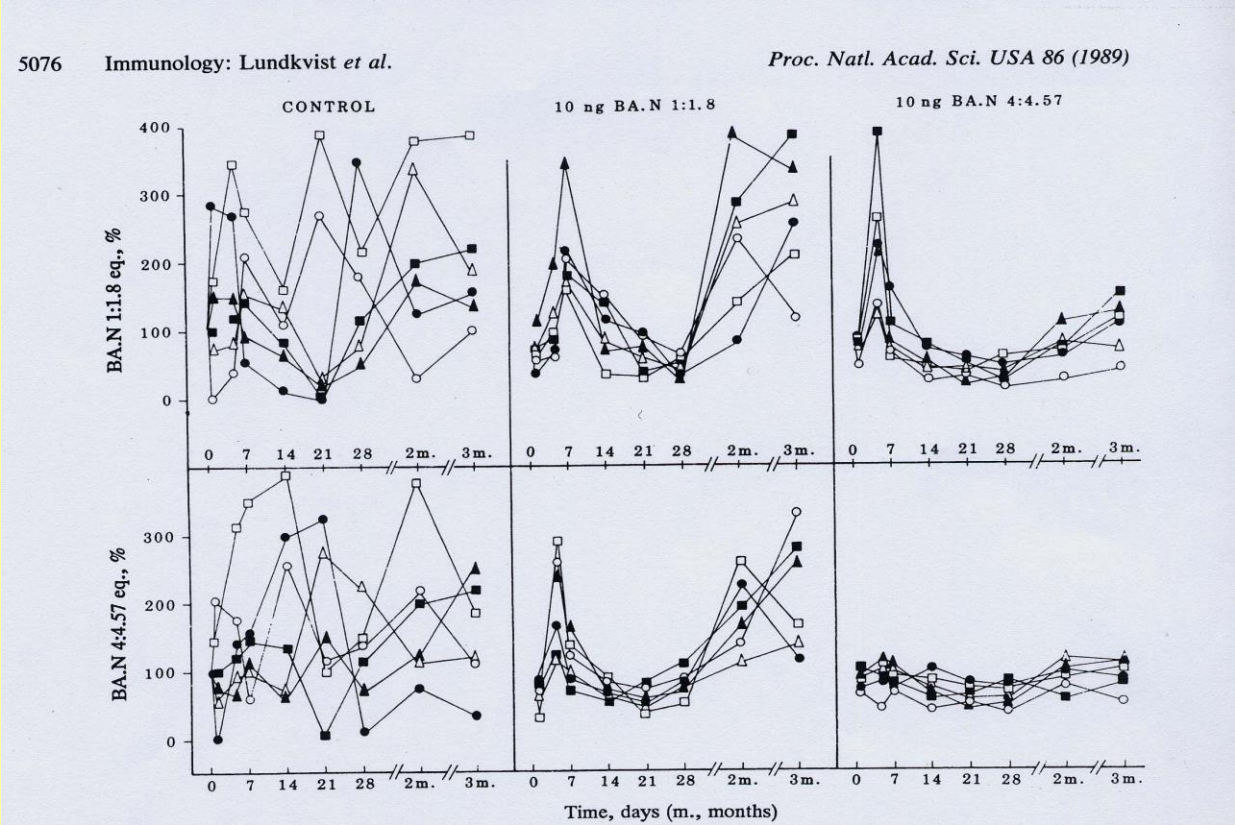
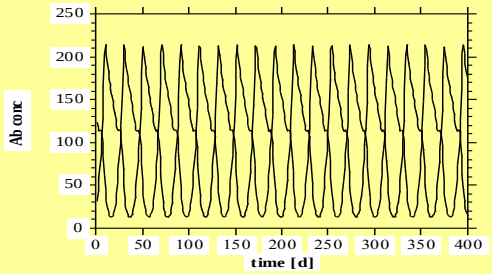
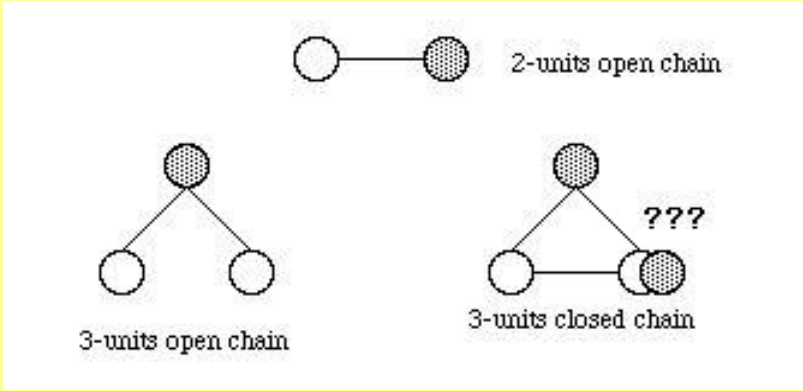


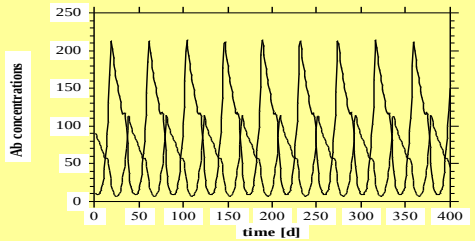
FIG. 2. Time course of expression of two connected idiotypes (BA.N 1:1.8 and BA.N 4:4.57) in adult BALB/c mice that had been injected with 10 ng of either idiotype or had been left untreated. Repeated bleedings from six individual mice, per group, at the indicated times were analyzed for the expression of BA.N 1:1.8 (*Upper*) and BA.N 4:4.57 (*Lower*) equivalents. The concentrations obtained on the day of the first bleeding (day 0) were normalized to 100% for each mouse, and all other titers are expressed as a percentage of this initial value.

Frustrated chaos in biological networks

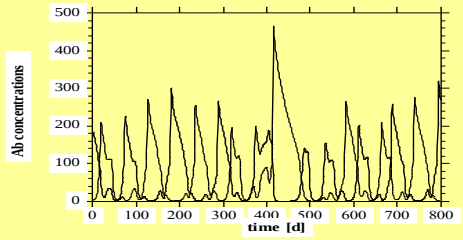


2-clone case $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



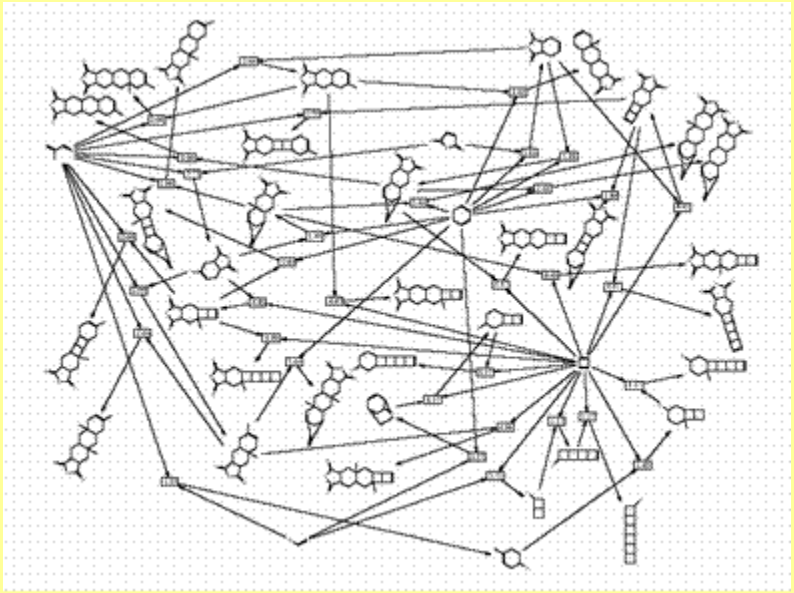


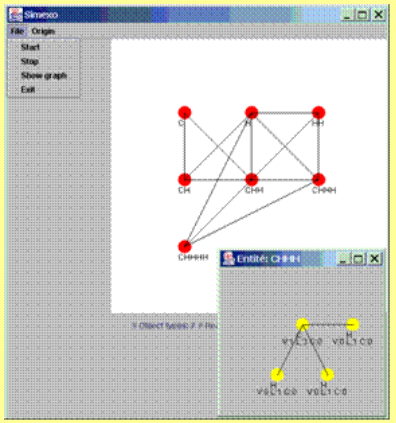
3-clone open chain $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

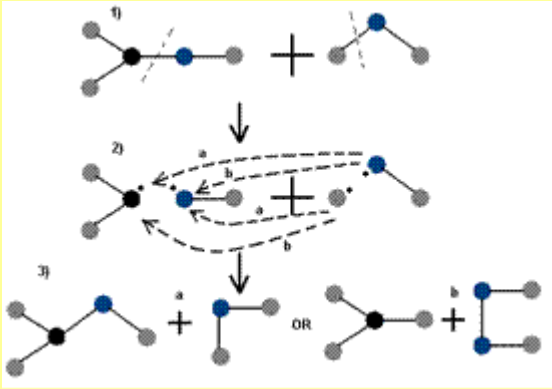


3-clone closed chain $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Network of chemical reactions

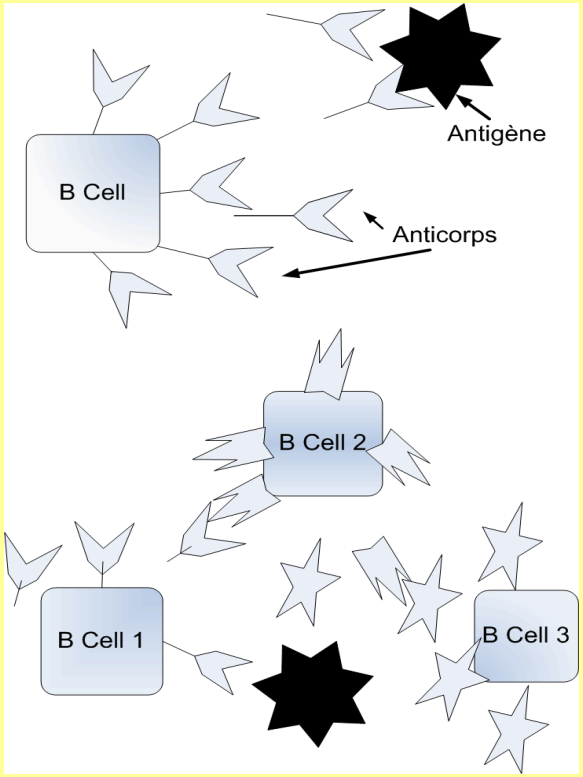


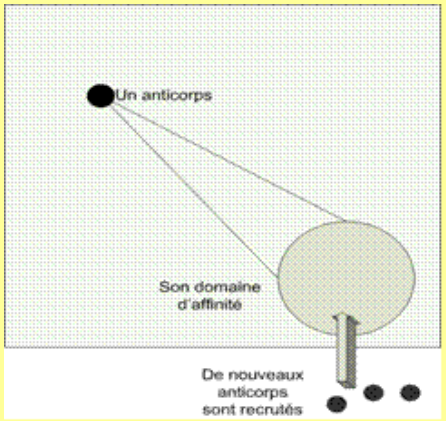


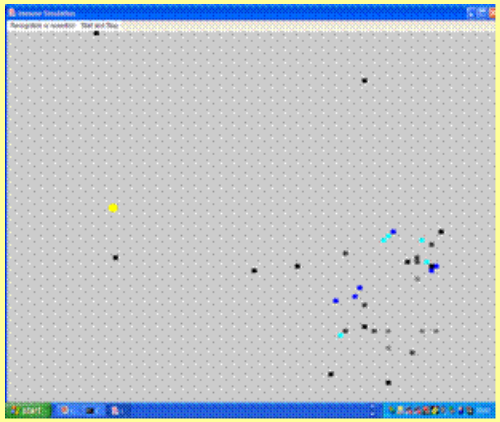


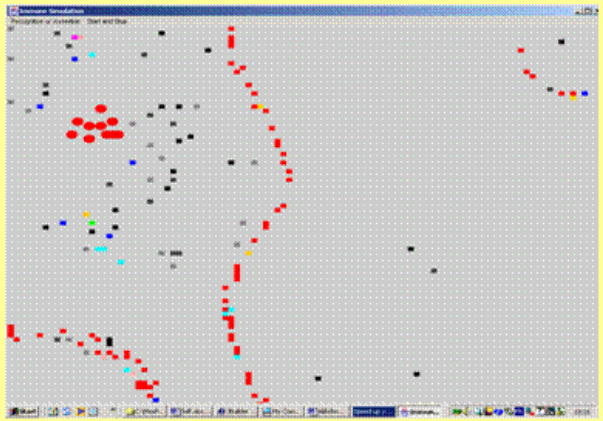
- Dynamics = kinetics
- Metadynamics = appearance and disappearance of molecules

Immune Networks

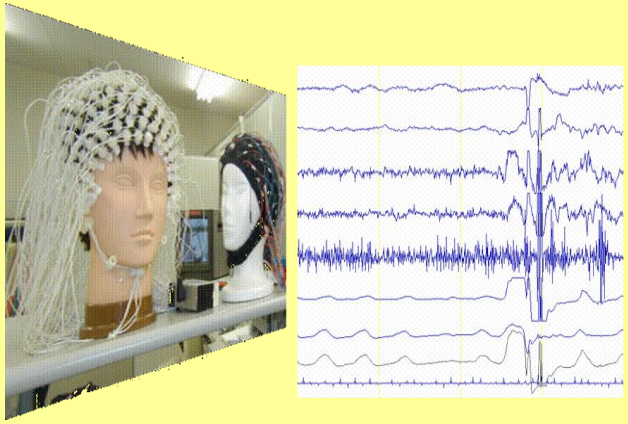


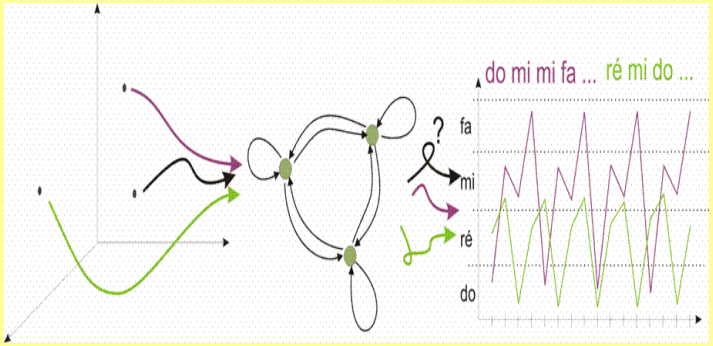


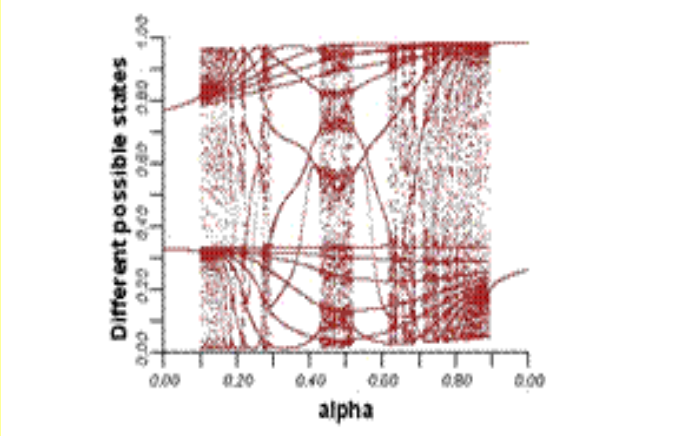


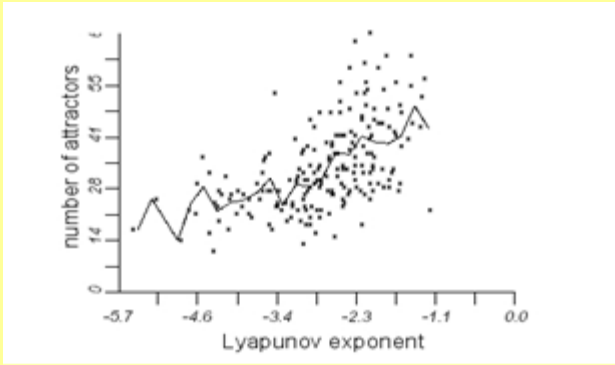


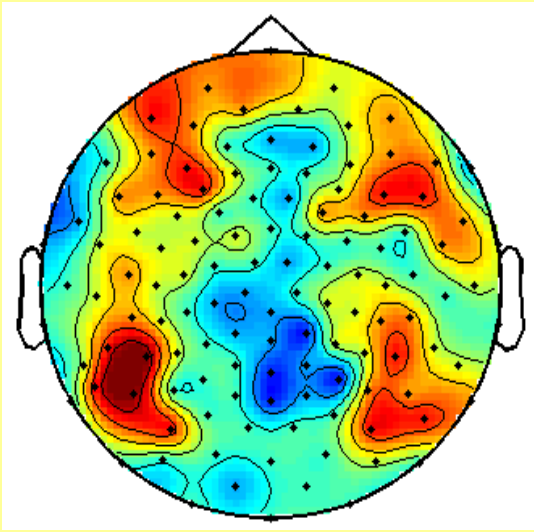
Neural Networks

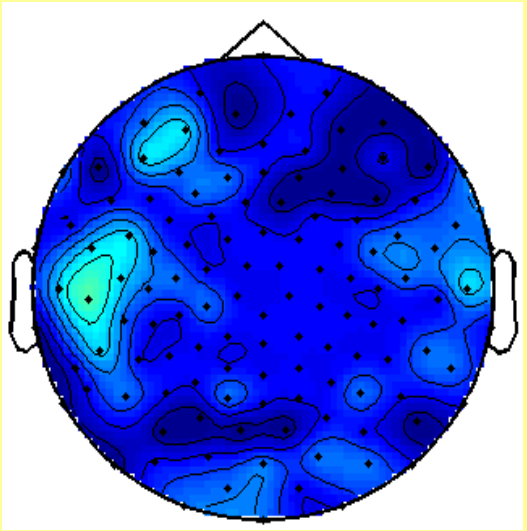






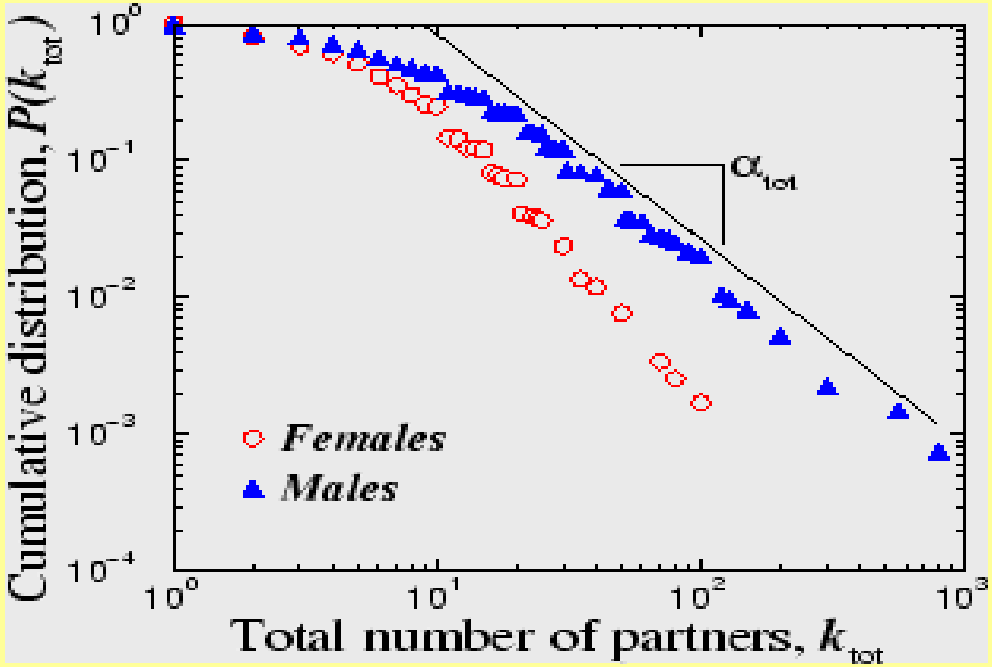


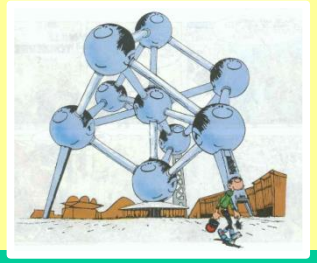




Elementary dynamics:propagation

Epidemic propagation





A new route to cooperation

With F. Santos - IRIDIA

The prisoner's dilemma

P1/P2	Cooperate	Compete
Cooperate	(1,1)	(-2,3)
Compete	(3,-2)	(-1,-1)

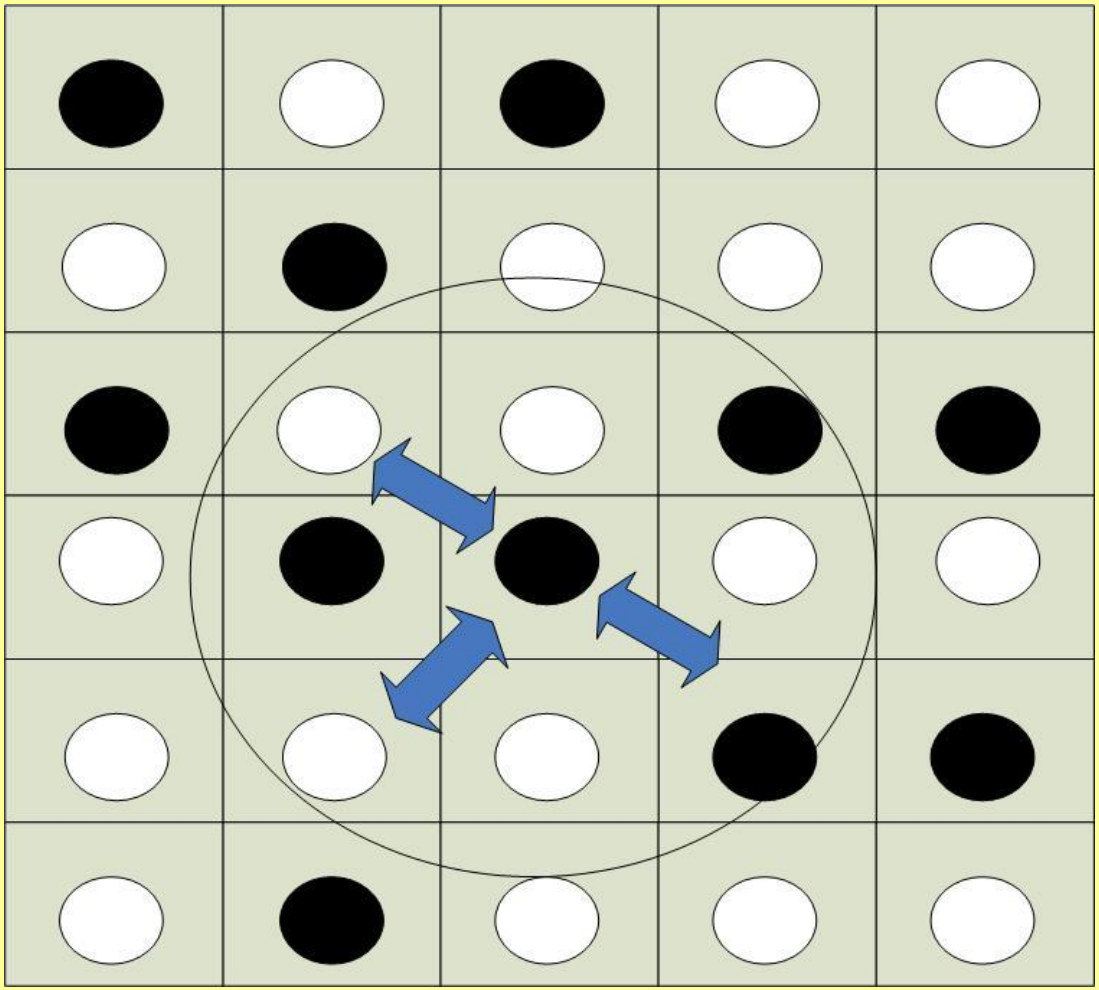
The winning strategy for both players is to compete. But doing so, they miss the cooperating one which is collectively better. The common good is subverted by individual rationality and self-interest.

But is competitive behaviour and collective distress avoidable ?

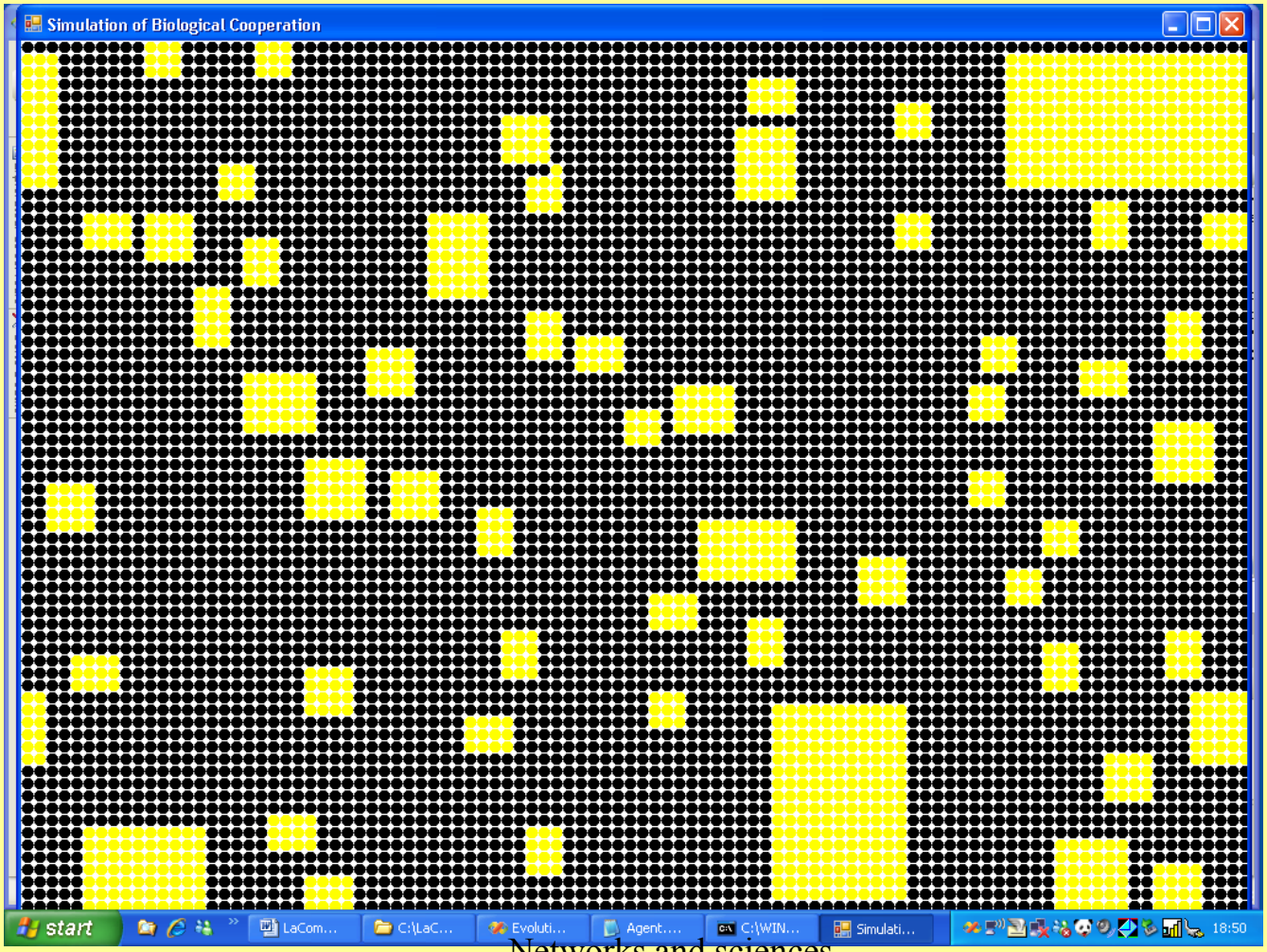
- So far the prisoner's dilemma is lacking some crucial quality that real world situations have.
- 1) Iterated version: play several moves and cumulate your reward over these moves.
- 2) Distribute spatially the players (CA): each cell just cooperates with its immediate neighbours and adapts the local best strategy. Cluster of nice individuals emerge and can prosper in hostile environments -> EVOLUTIONARY GAME THEORY

The spatial cellular automata simulation

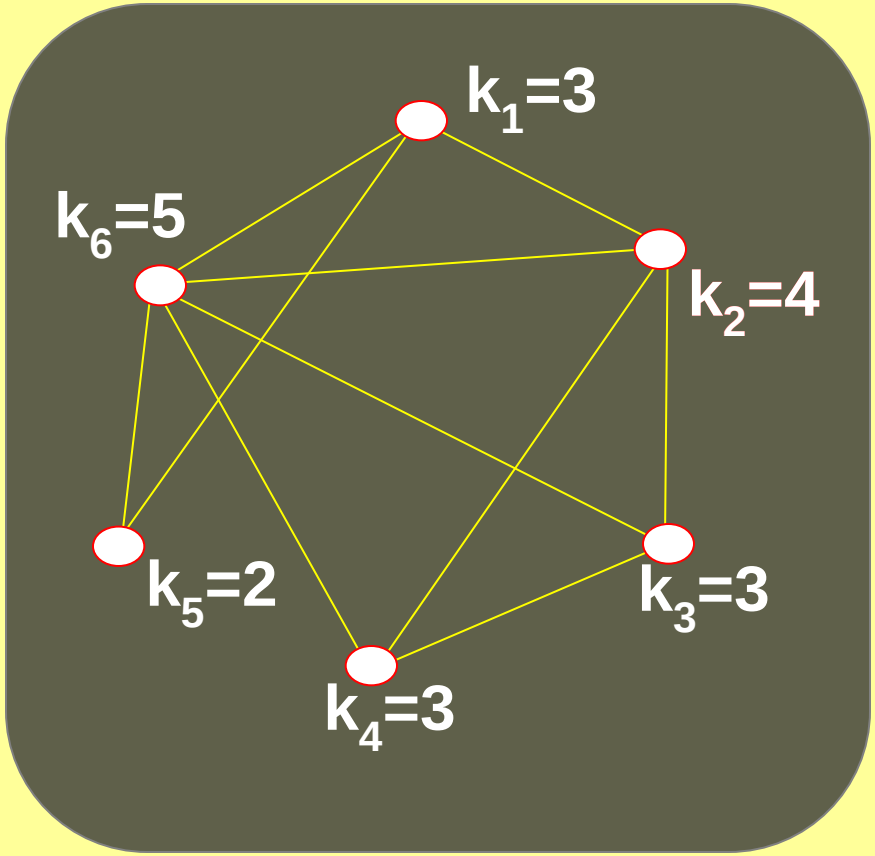
- Largely inspired by Nowak's work on spatial prisoner dilemma
- A cellular automata in which every cell contains one agent (specialist or generalist)
- In all cells, asynchronously, an agent will subsequently:
  - interact with its neighbors (Moore neighborhood) to "consume" them.
    - Sum the payoff according to the payoff matrix
  - replicate
    - Adopt the identity of the fittest neighbor
- For a given number of iteration steps



Nowak's cooperators vs defectors



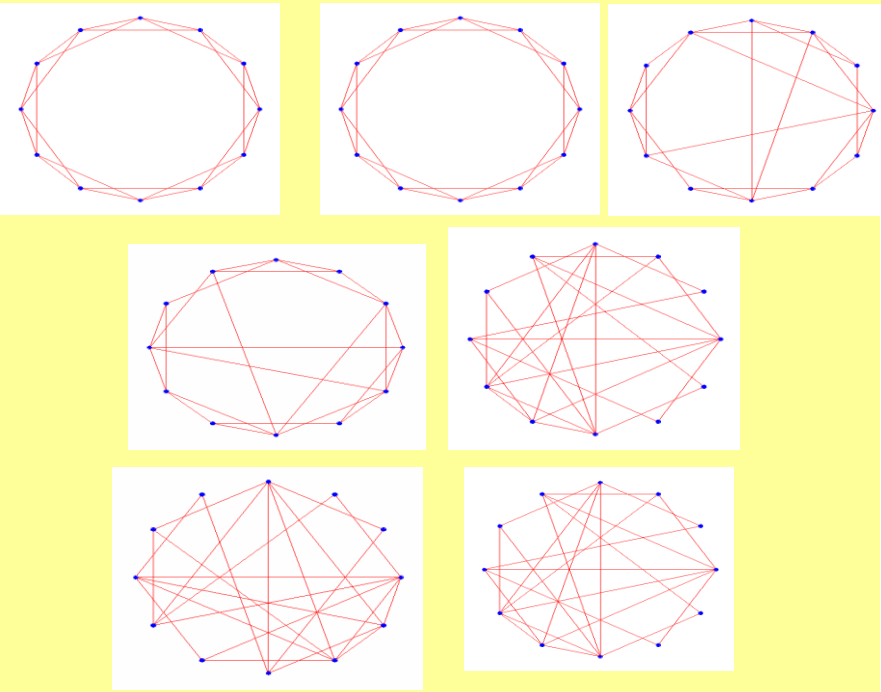
Setting the stage

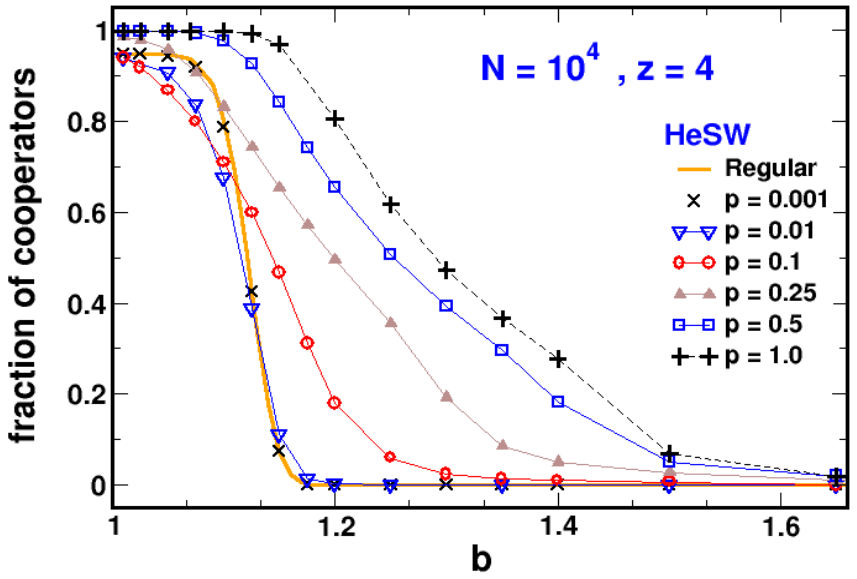


- Stochastic replicator dynamics:
  - Vertex x plays k_x times per generation and accumulates payoff f_x.
  - Choose a random neighbor y with payoff f_y.
  - Replace strategy m_x by m_y with probability:

$$p = \max\left[0, \frac{f_x - f_y}{k_> (T - S)}\right]$$

Games on graphs

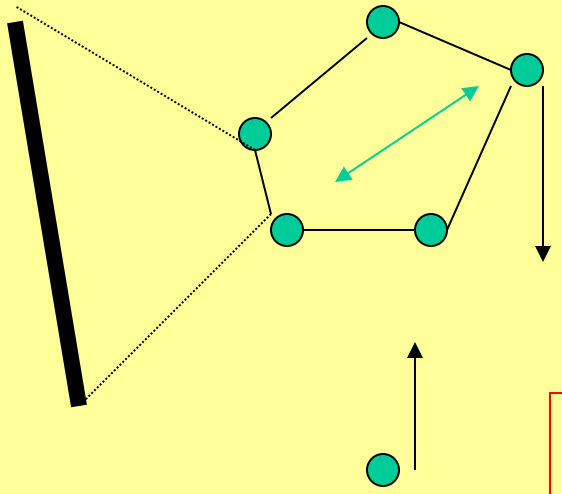




- Conclusions:
  - The more heterogeneous, the more cooperative.
  - Cs benefit most from heterogeneity.

Plastic networks: parametrically and structurally: Network Metadynamics

- Various dynamical changes, that Varela called: dynamics and metadynamics

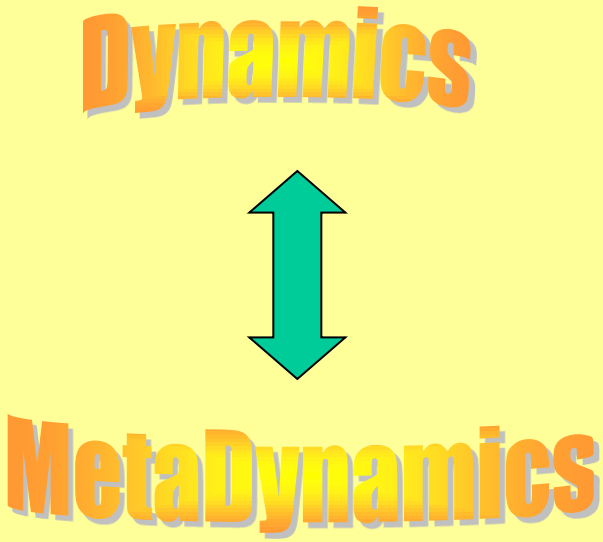


- modification of connexions
- addition of connexions
- addition of new nodes
- suppression of existing nodes

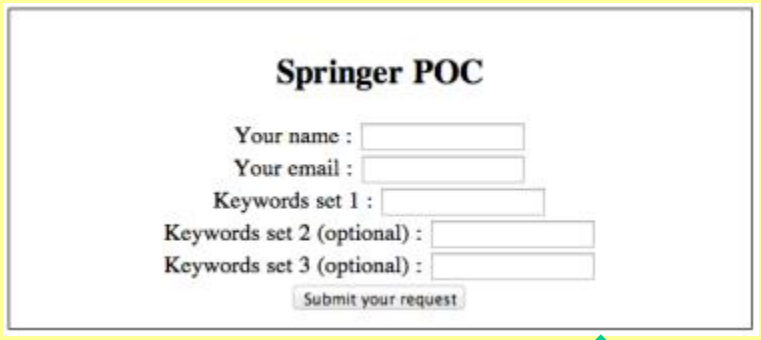
The organisation is maintained independently of the constituants

- This is the case for neuro, immuno, chemical, sociological networks, PC networks

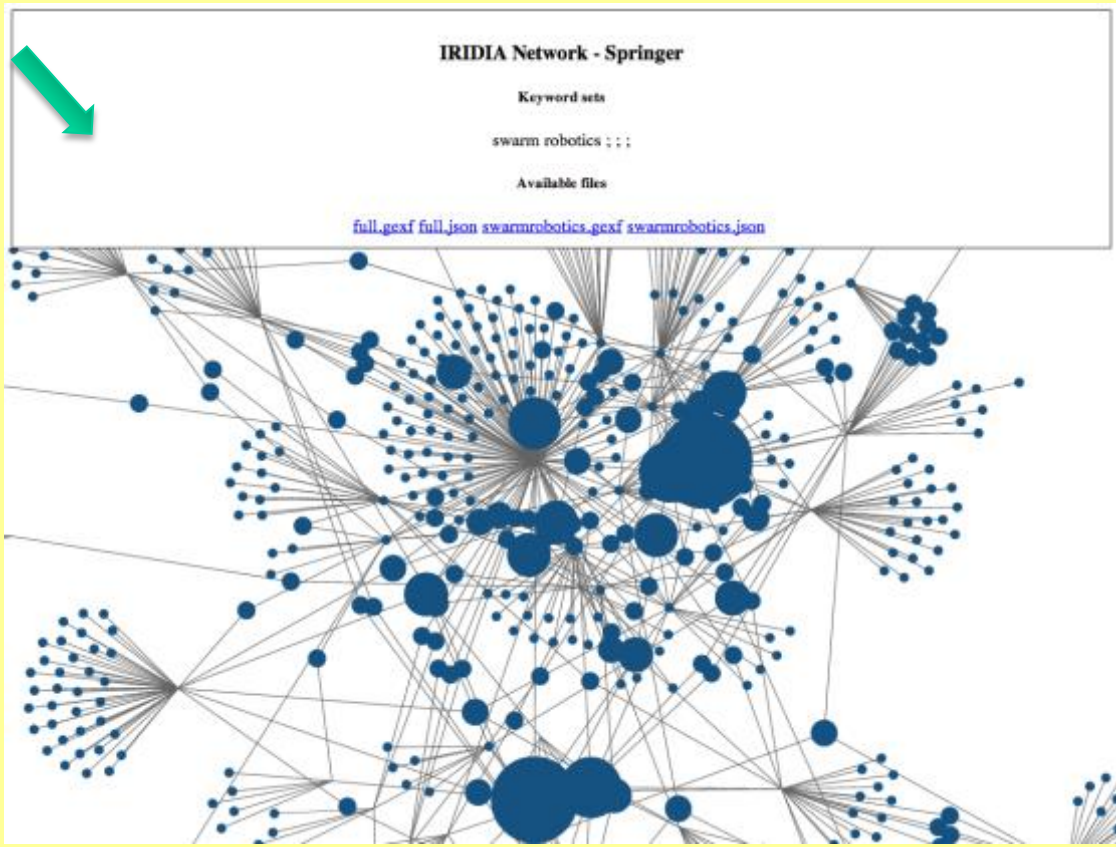
A key interdependency



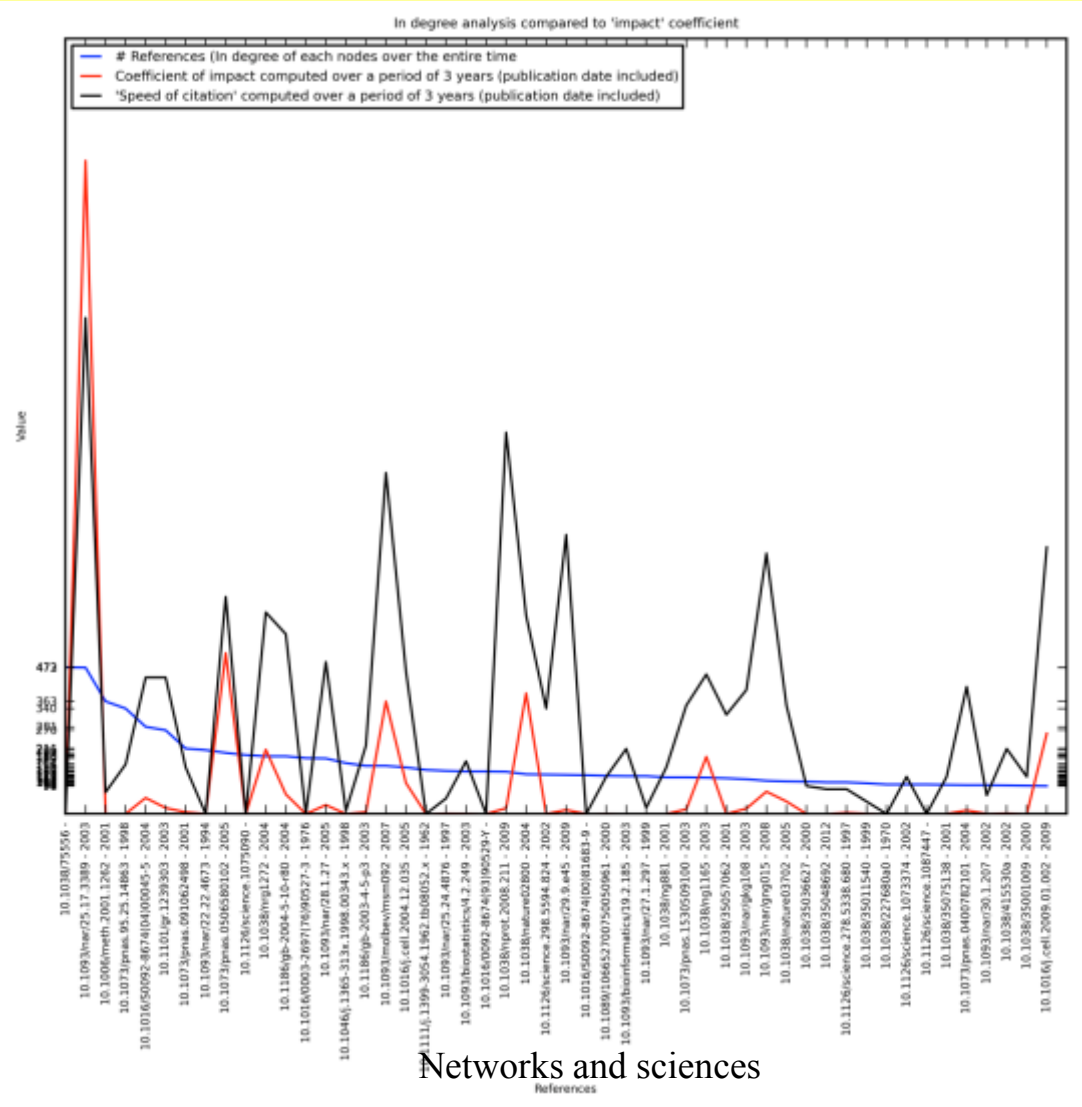
Springer Link : With networks ?



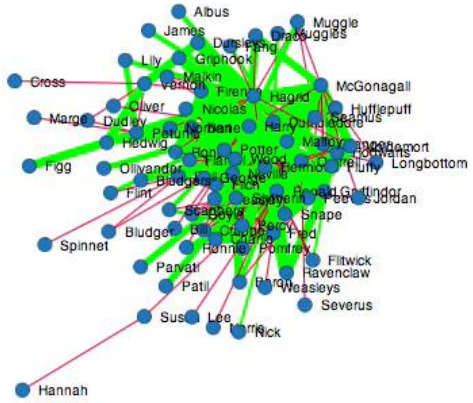
- Crawler
- How ?
  - Python script
  - HTTP Requests
- What ?
  - DOI
  - References
  - Date



Results

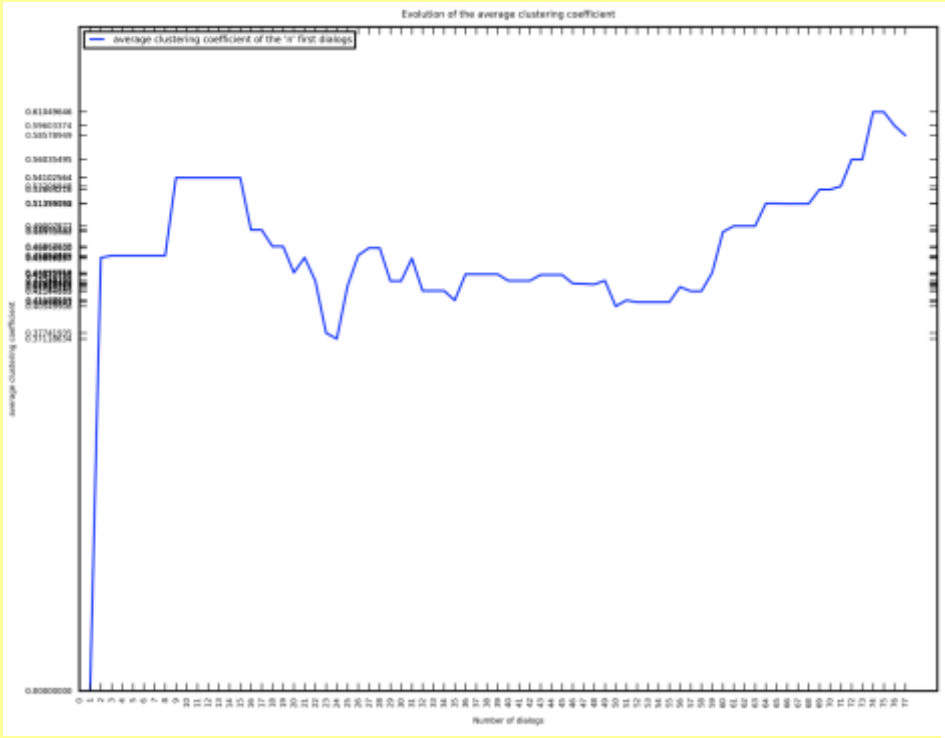

In degree analysis compared to 'impact' coefficient
References (In degree of each nodes over the entire time
Coefficient of impact computed over a period of 3 years (publication date included)
'Speed of citation' computed over a period of 3 years (publication date included)
Value
472
References

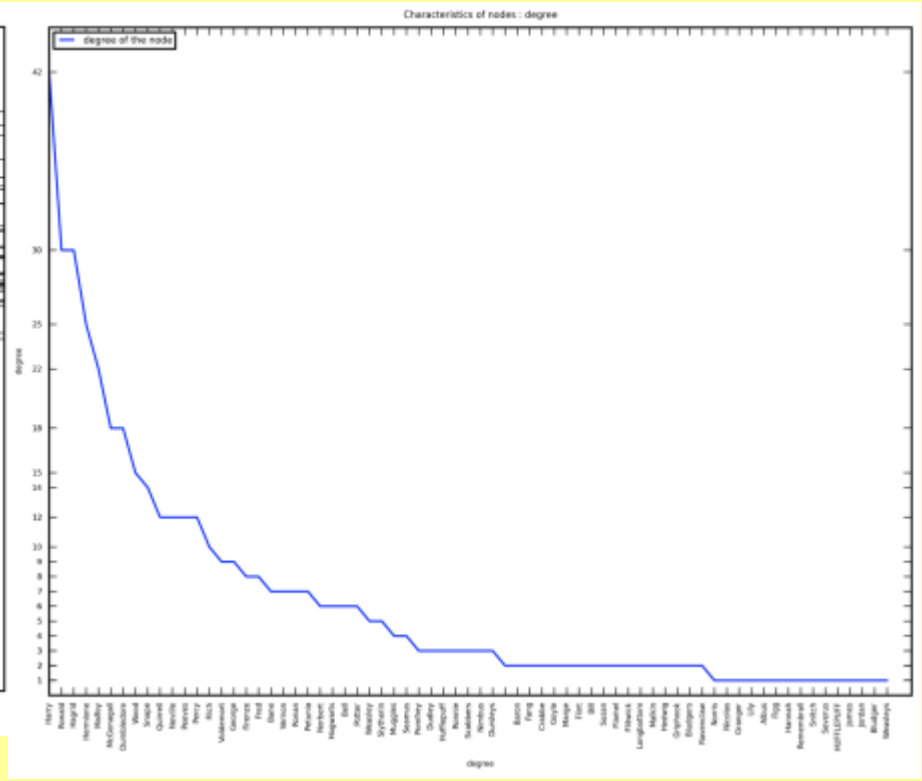
Book analysis



Book analysis

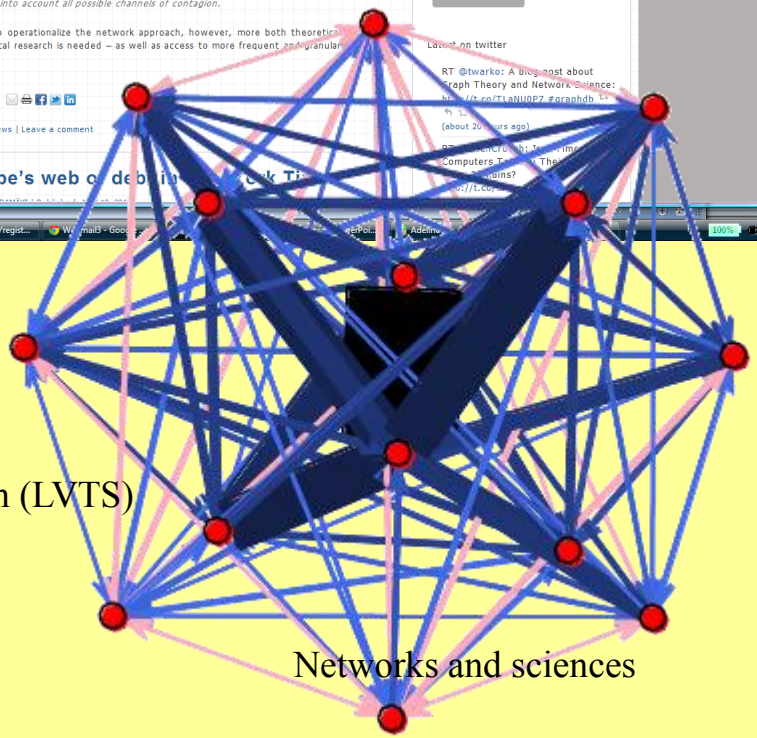
- Clustering coefficient and Degree





Systemic contagion in financial systems

Financial Network

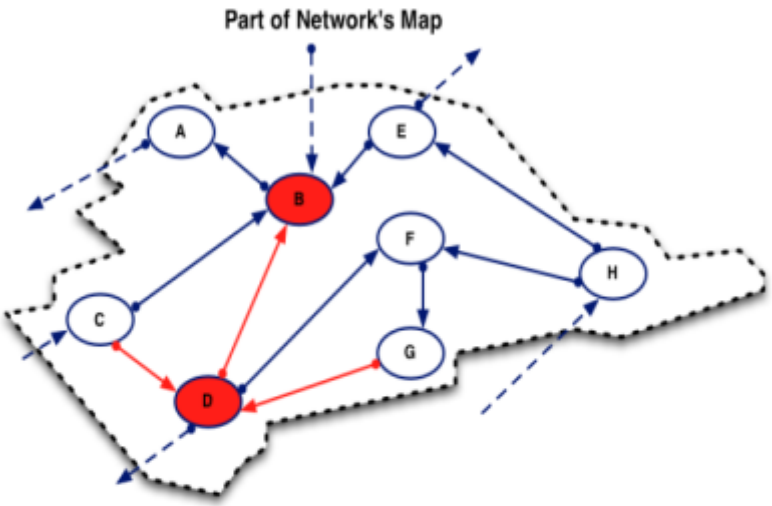


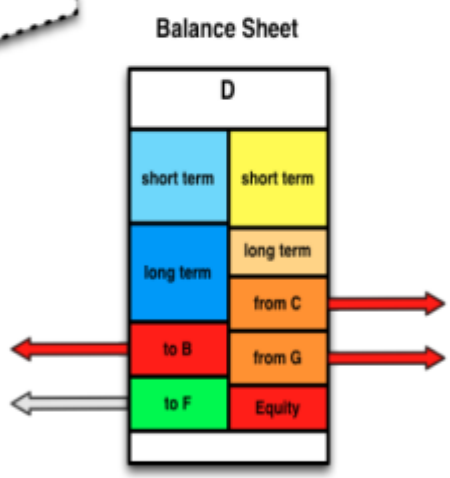
Intraday network structure
of payment activity in
the Canadian
Large Value Transfer System (LVTS)

Liasons Dangereuses:
Increasing connectivity,
risk sharing and systemic risk
(by Stefano Battiston,
Domenico Delli Gatti,
Mauro Gallegati,
Bruce C. Greenwald and Joseph E. Stiglitz)

The Model

Contagion Channels - First type

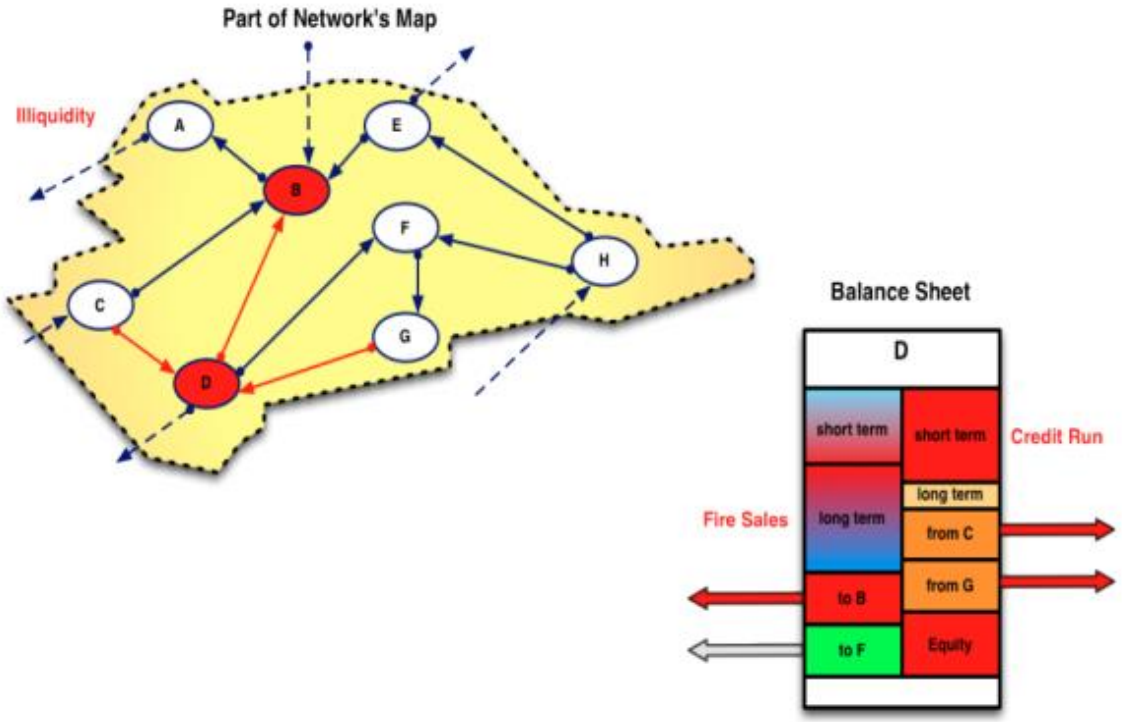




$$\eta_i(t) = \eta_i(0) - \frac{k_{fi}(t)}{k_i}$$

The Model

Contagion Channels - Second type



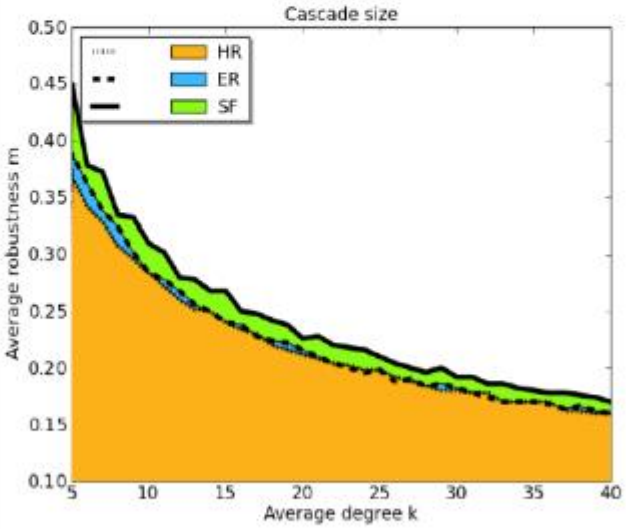
$$\eta_i(t) = \eta_i(0) - \frac{k_{fi}(t)}{k_i} - b, \qquad \text{if } \eta_i(0) < \gamma k_{fi}(t)$$

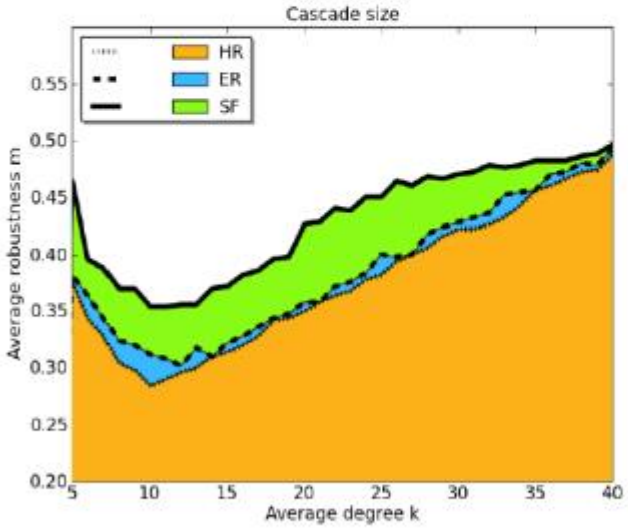
Experiments

Imposing the out degree distribution

Scenario Random individual financial robustness

Result Scale Free is more fragile in illiquid markets





Conclusions: networks are everywhere

Facebook révèle que le monde est plus petit qu'on le pense

Le développement du réseau social sur internet Facebook a conduit à réviser la théorie selon laquelle il existerait "six degrés de séparation" entre tous les individus, une étude de l'Université de Milan ayant déterminé que le site avait ramené ce chiffre à 4,74.

"Le monde est encore plus petit que nous le pensions", écrivent cinq chercheurs (Lars Backstrom, Paolo Bodli, Marco Rosa, Johan Ugander, Sebastiano Vigna) dans une étude référencée par Facebook, qui y a collaboré.

Selon une hypothèse formulée pour la première fois dans les années 1920, n'importe quel individu peut être relié à n'importe quel autre par une chaîne de relations individuelles de six personnes.

Cette théorie s'appuie sur les travaux de Stanley Milgram et Jeffrey Travers, qui avaient demandé dans les années 1960 à 300 personnes vivant dans le Nebraska (centre des Etats-Unis) de faire parvenir une lettre à quelqu'un à Boston (Massachusetts, nord-est) par l'intermédiaire de connaissances. Un ami représentait un degré de séparation, l'ami d'un ami deux degrés, etc... Les lettres parvenues à leur destinataire avaient franchi en moyenne 6,2 degrés de séparation. La théorie des six degrés n'a jamais été considérée comme scientifiquement valable.

L'étude ayant bénéficié du concours de Facebook montre quant à elle que 99,6% des utilisateurs du site peuvent se connecter avec un autre internaute, via des connaissances, en cinq étapes étapes seulement, et 92% en quatre étapes.