INFO-F-409
Learning dynamics

An introduction to Game Theory

ULB

T. Lenaerts and Y.-M. De Hauwere
MLG, Université Libre de Bruxelles and
AI-lab, Vrije Universiteit Brussel

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Computational Game Theory

An introduction to Game Theory

Vrije Universiteit Brussel

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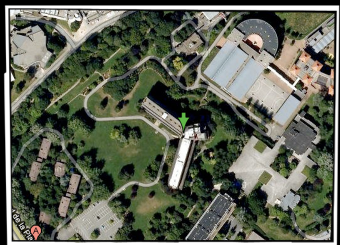
My coordinates



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Vrije Universiteit Brussel ULB MLG

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Schedule

Date	Description
18/09/2014	No course this day
25/09/2014	Game theory basics
2/10/2014	Mixed strategies and Nash algorithms
9/10/2014	Extensive form games and their equilibria
16/10/2014	Evolutionary game theoy
23/10/2014	Evolution of cooperation
30/10/2014	N-armed bandits (stateless reinforcement learning)
6/11/2014	Graphical games
13/11/2014	Reinforcement learning and MDPS
20/11/2014	No course this day
27/11/2014	Sparse Interactions
4/12/2014	Project preparation time
11/12/2014	Selfish load balancing
18/12/2014	
25/12/2014	Winter break
1/01/2015	
Exam: Article + presentation of group project	

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Practical things

- ~3 Assignments during the course
  - They are taken into account (50%) for the final grade.
  - Assignments are personal (NO TEAMWORK), this will be checked !
  - Mail your solutions
    - NO paper copies !!!
    - Please provide a single (self-contained) *.PDF file.
  - Schedule (temporary)
    - Assignment 1 (presentation: 10.10.2012) Game theory basics
    - Assignment 2 (presentation:??.??.2012) Evolutionary game theory
    - Assignment 3 will be provided by Prof Nowé

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Practical things

- **Exam = scientific project**
  - study a topic related to the course (some possibilities will be provided)
    - Look for something **YOU** like on for instance google scholar
  - Formulate a question you want to study
  - implement a software that allows you to answer that question
  - Write a scientific article (The unofficial guide for authors)
  - Present and **discuss** articles in January

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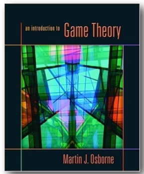
Bibliography



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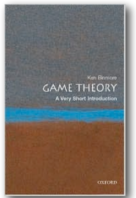

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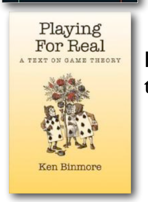

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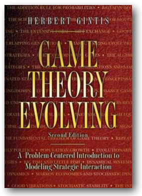

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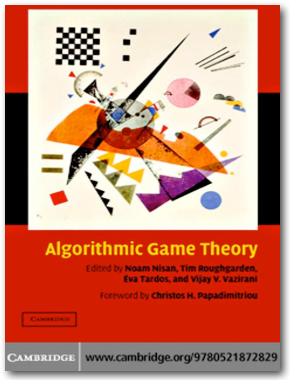
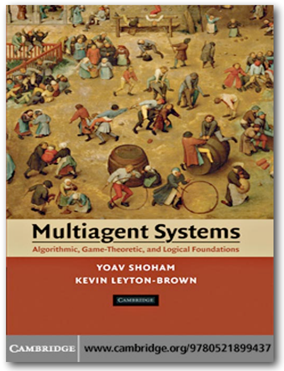
for computer science





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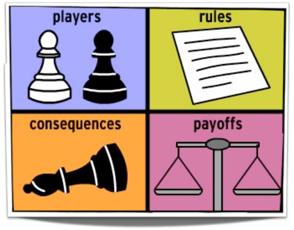
What?

[...] *A game is a competitive activity in which players contend with each other according to a set of rules* [...]



[...] *Game theory is a theory/tool that helps us understand situations in which decision-makers interact* [...]

What ?

games

What ?

economy

What ?

economy

What ?

g

economy

10-4

What ?

gam

politics

10-5

What ?

Game shows

10-6

What?

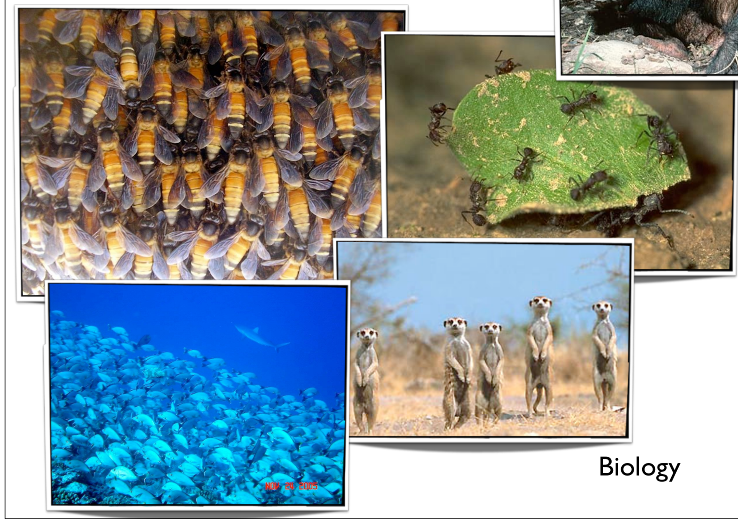
Fragment from Golden Balls (ITV1)

11-1

What?

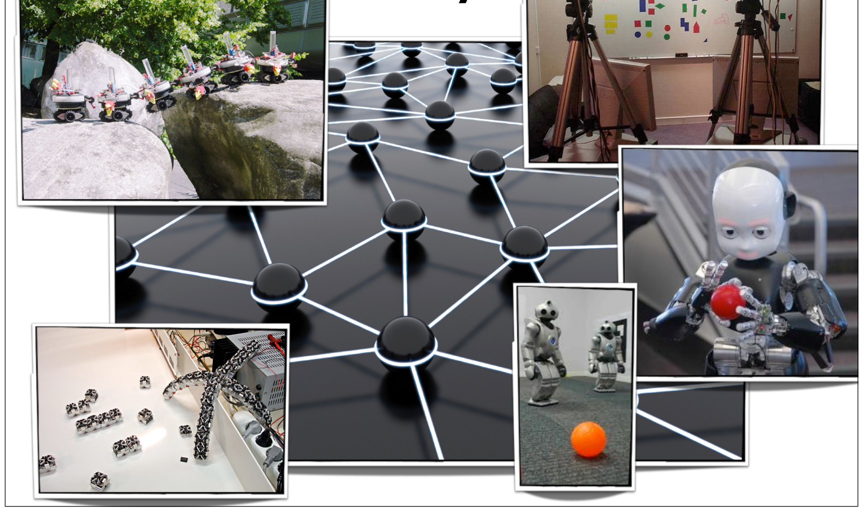
Fragment from Golden Balls (ITV1)

What ?



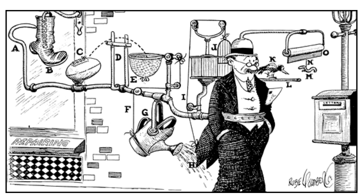
Biology

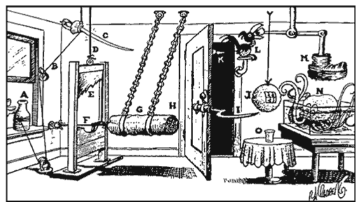
Why?

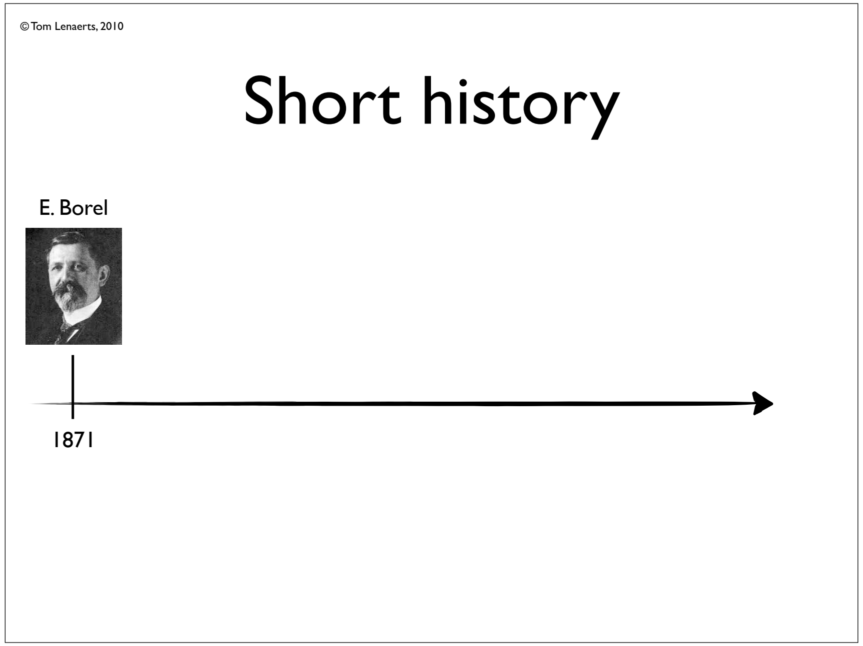


Model building

[...] *Game-theoretic modeling starts with an idea related to some aspect of interacting decision-makers. We express this idea precisely in a model,* ***incorporating features*** *of the situation that appear to be* ***relevant***. [...] *We wish to put enough ingredients into the model to obtain* ***nontrivial insights***, [...] *we wish to* ***lay bare the underlying structure*** *of the situation as opposed to describing its every detail. The next step is to analyze the model - to discover its* ***implications*** [...] *Our analysis may* ***confirm our idea, or suggest it is wrong****. If it is wrong the analysis should help us understand why* [...]







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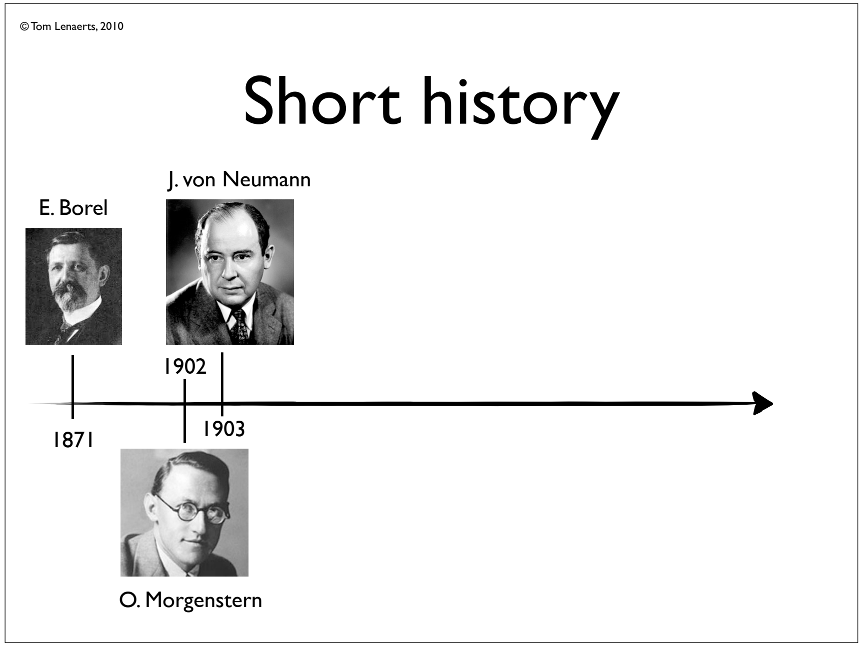

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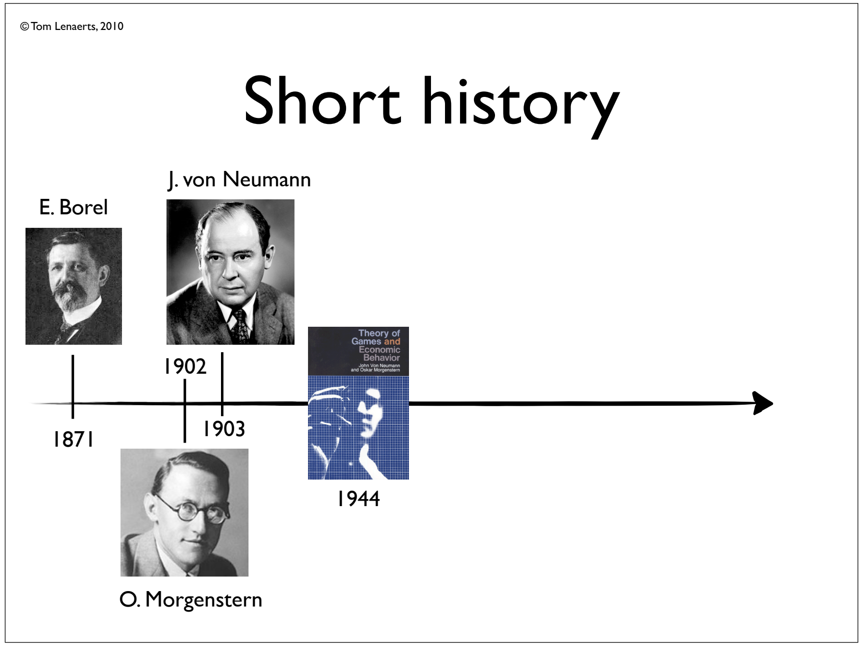

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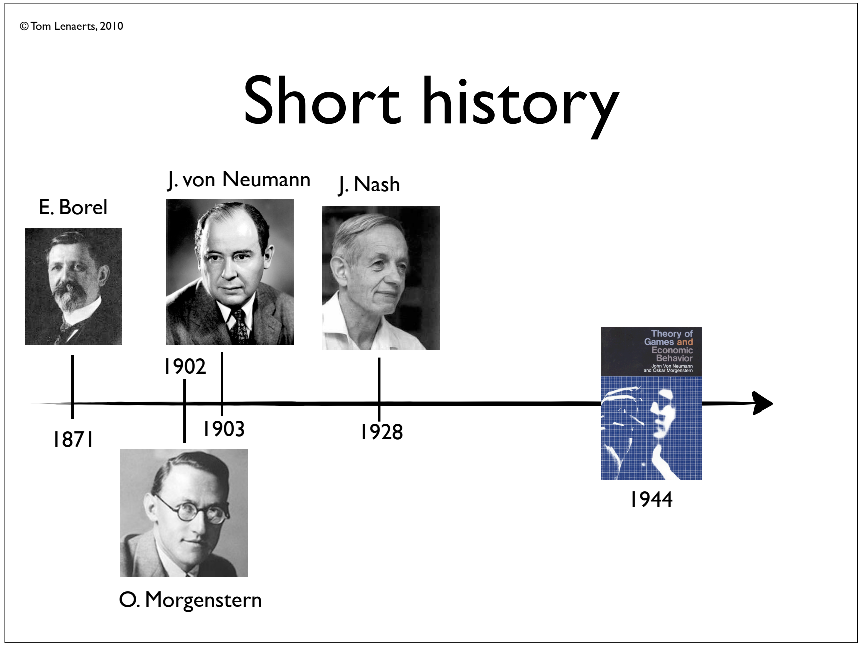

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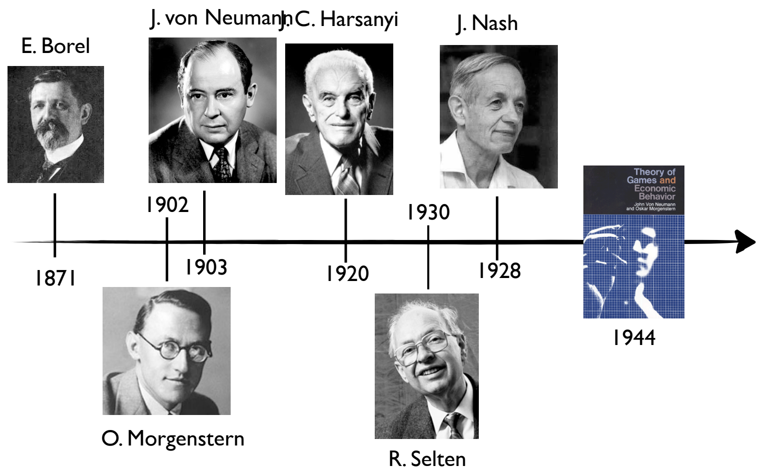
Short history



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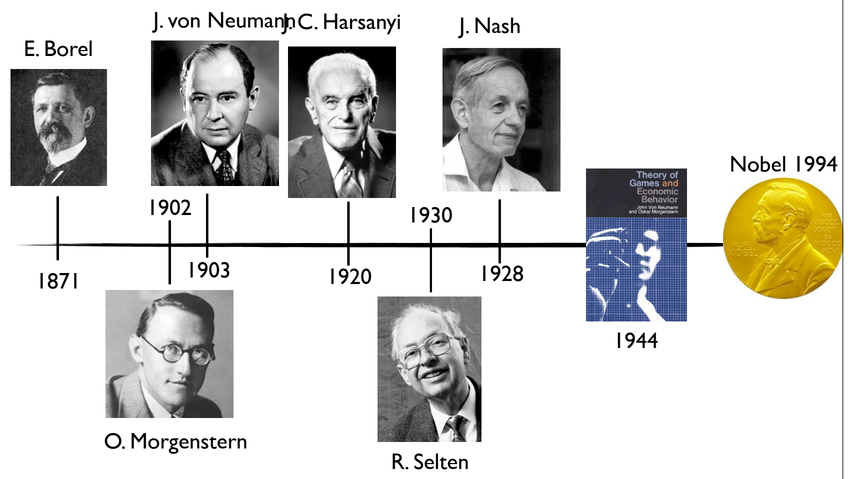
Short history



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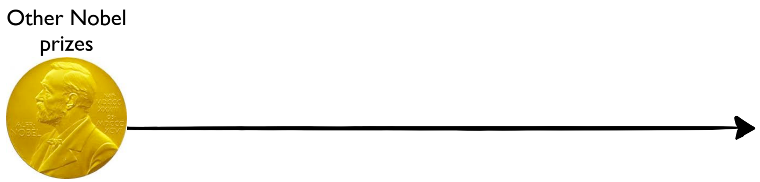
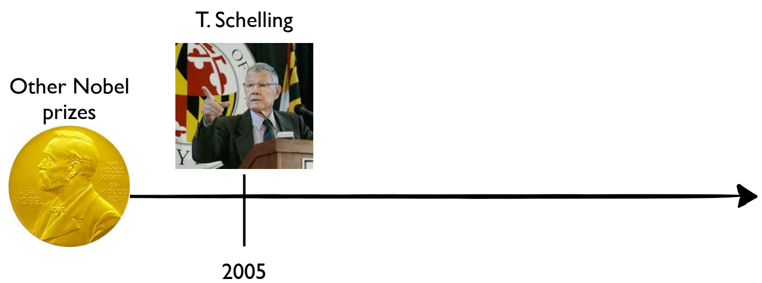
recent history



16-1

recent history



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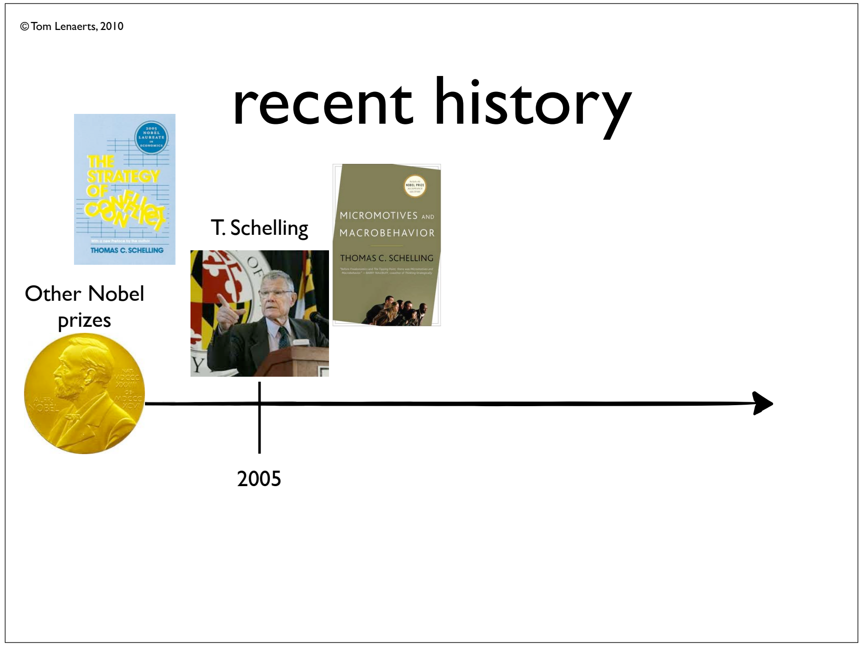

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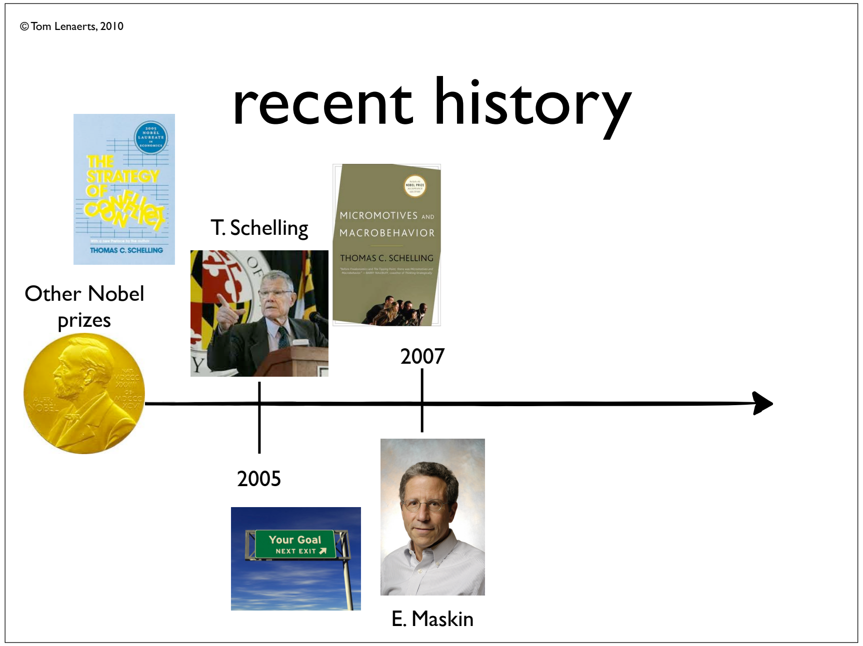

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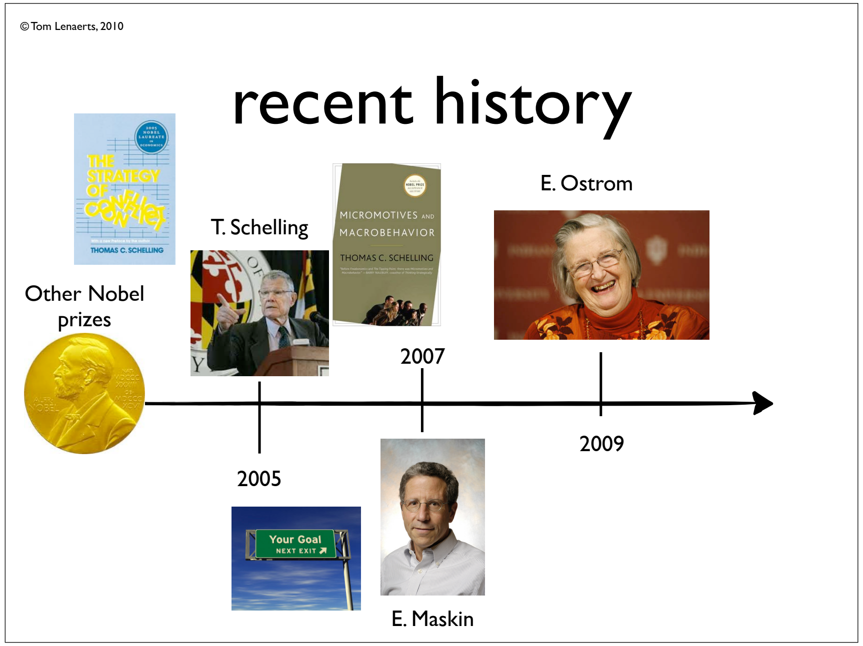

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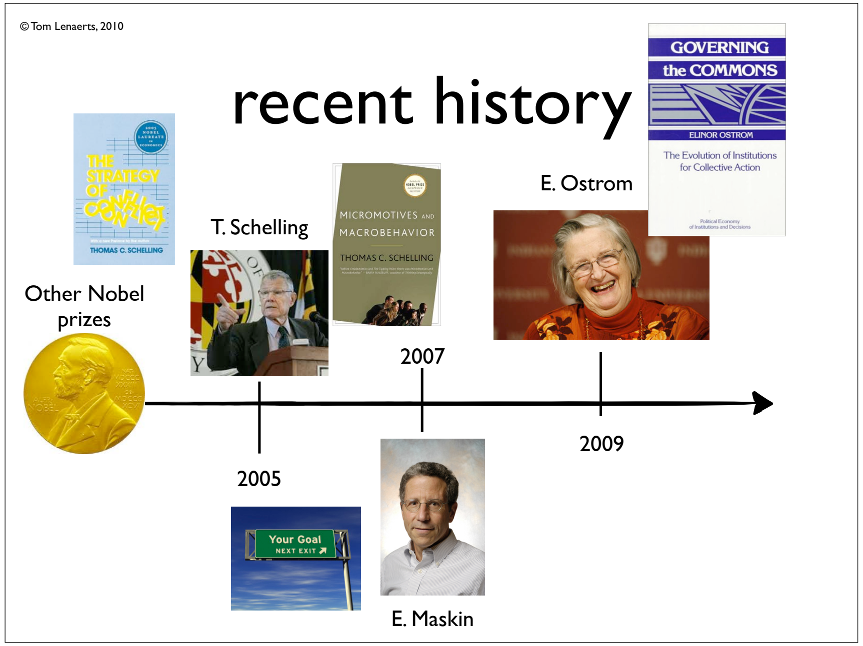
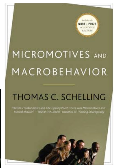


16-6

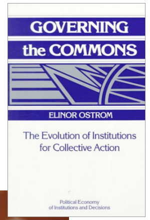
recent history

Other Nobel prizes

T. Schelling

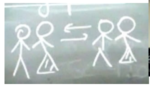


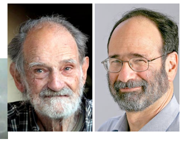
E. Ostrom



2005 2007 2009 2012

E. Maskin





L. Shapley and A.E Roth

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Rational choice

A decision-maker chooses the best **action** according to her **preferences**, among all the actions available

Actions in Golden Balls game : $A=\{split,steal\}$

Preferences should be consistent and can be represented by a function $u(x)$

In Golden Balls game : $u(steal) > u(split)$

The scale of he numbers in this function do not relate to the importance of a preference

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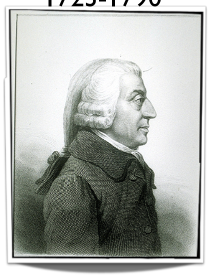
The theory of rational choice

A. Smith
1723-1790



[...] *The action chosen by a decision-maker is at least as good, according to her preferences, as every other available action* [...]

This theory pervades economic theory !

Is not always applicable !

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Rational choice according to Nash

Fragment from A Beautiful mind (2001)

19-1

Rational choice according to Nash

Fragment from A Beautiful mind (2001)

Other decision-makers

A decision-maker preferences' are affected by the preferred actions of other decision-makers

Such situations are modeled as games !

Strategic games

Fragment fromThe Big-Bang Theory (2008)

Consists of :

- a set of players
- for each player a set of actions
- for each player, **preferences over the set of actions**

Strategic games

Fragment fromThe Big-Bang Theory (2008)

Consists of :

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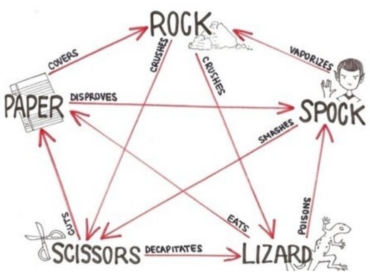
Strategic games

Players : Sheldon and Rajesh

Actions : {rock, paper, scissors, lizard,spock}

Preferences :



©Tom Lenaerts, 2010

Strategic games

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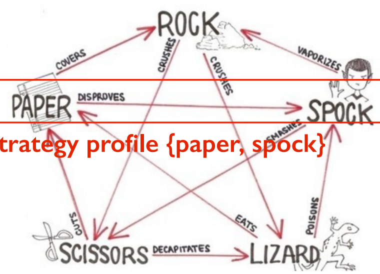
Actions : {rock, paper, scissors, lizard,spock}

Preferences :



strategy profile {paper, spock}

©Tom Lenaerts, 2010

The Golden Balls dilemma

Players : Sarah and Steve

Actions : {split, steal}

Preferences :

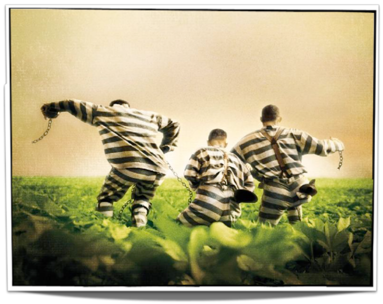
	split	steal
split	50075 / 50075	100150 / 0
steal	0 / 100150	0 / 0

©Tom Lenaerts, 2010

The prisoner's dilemma

Players : Two thiefs

Actions : {Quiet, Fink}



Preferences :

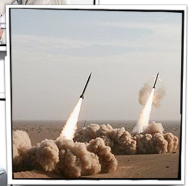
	Quiet	Fink
Quiet	3 / 3	7 / 0
Fink	0 / 7	1 / 1

$u(Fink,Quiet) > u(Quiet,Quiet) > u(Fink,Fink) > u(Quiet,Fink)$

The prisoner's dilemma

This game extends to a variety of situations

- working on a joint project,
- duopoly
- arms race
- use of a common property



The chicken game

Fragment from Footloose (1984)

The chicken game

Fragment from Footloose (1984)

The chicken game

Players : Kevin Bacon and Chuck

Actions : {swerve, straight}

a.k.a. snowdrift game

	swerve	straight
swerve	0 / 0	+1 / -1
straight	-1 / +1	-10 / -10

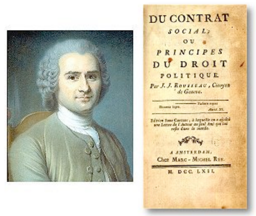
u(straight,swerve) > u(swerve,swerve) > ***u(swerve,straight)*** > ***u(straight,straight)***

The stag-hunt game

The stag-hunt game

Players : two hunters

Actions : {whale, fish}



a.k.a. coordination game

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

u(whale,whale) > u(fish,whale) *> u(fish, fish) > u(whale, fish)*

arms race?

	Refrain	Arm
Refrain	2, 2	0, 3
Arm	3, 0	1, 1

prisoner's dilemma

	Refrain	Arm
Refrain	3, 3	0, 2
Arm	2, 0	1, 1

stag-hunt game
a.k.a. security dilemma

Matching pennies

Strictly competitive

Zero-sum game

Players : two players

Actions : {head, tail}

	head	tail
head	+1, -1	-1, +1
tail	-1, +1	+1, -1

Matching pennies

Example : IPad and look-a-likes

A newcomer will prefer that his ipad-clone looks and feels like the original

The established producer wants to ensure the difference

© Tom Lenaerts, 2010

Matching pennies

Example : IPad and look-a-likes

A newcomer will prefer that his ipad-clone looks and feels like the original

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© Tom Lenaerts, 2010

Bach-Stravinsky game

a.k.a. Battle of the sexes

Players : two players

Actions : {Bach, Stravinsky}

	Bach	Strav.
Bach	1 2	0 0
Strav.	0 0	2 1

Asymmetric games

A game is called symmetric when the row and column player have the same preferences over the same actions

... when they have the same payoff matrix ($A=B^T$)

Symmetric games : prisoners dilemma, the chicken game, the stag-hunt game, ...

Asymmetric games : Bach-Stravinsky, inspection game, ...

Inspection game

"A tax authority wants taxpayers to truthfully report income, an employer wants an employee to work hard, a regulator wants a factory to comply with pollution regulations, police want motorists to observe speed limits, etc.

A fundamental problem for authorities is how to induce compliance with desired behavior when individuals have incentives to deviate from such behavior. A standard approach is to monitor a proportion of individuals and penalize those caught misbehaving." (Quote from D. Nosenzo et al 2010 Discussion Paper 2010-

	Comply	Cheat
Don't Inspect	60, 25	0, 40
Inspect	52, 25	12, 20

Symmetricalization

An asymmetric game can be transformed into a symmetrical version of the game either by

1. Assuming that each player can act as row and column player 50% of the time

DC meets DC
i) D agains C = 60
ii) C against D = 25

average = 42.5

	DC	DH	IC	IH
DC	42.5, 42.5	12.5, 50	42.5, 37.5	12.5, 46
DH	50, 12.5	20, 20	40, 18.5	10, 26
IC	37.5, 42.5	18.5, 40	37.5, 37.5	18.5, 36
IH	46, 12.5	26, 10	36, 18.5	16, 16

Symmetricalization

An asymmetric game can be transformed into a symmetrical version of the game either by

2. Assuming that both players have the same actions but only receives payoff when playing against the correct ones

When playing D against D, I against I, C against C and H against H, there is no payoff

	D	I	C	H
D	0, 0	0, 0	60, 25	0, 40
I	0, 0	0, 0	52, 25	12, 20
C	25, 60	25, 52	0, 0	0, 0
H	40, 0	20, 12	0, 0	0, 0

Nash equilibrium

Which action will be chosen by each player?

Nash equilibrium

Which action will be chosen by each player?

Theory of rational choice states that each player chooses the **best available action**

Nash equilibrium

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Theory of rational choice states that each player chooses the **best available action**

Since this choice depends on the actions of the other player, each player must form a **belief** about the other players' actions and preferences

Nash equilibrium

Which action will be chosen by each player?

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Since this choice depends on the actions of the other player, each player must form a **belief** about the other players' actions and preferences

This belief is formed based on the **knowledge of the game and past experiences**

Nash equilibrium

Which action will be chosen by each player?

Theory of rational choice states that each player chooses the **best available action**

Since this choice depends on the actions of the other player, each player must form a **belief** about the other players' actions and preferences

This belief is formed based on the **knowledge of the game and past experiences**

BUT ! each play is considered in isolation (players do not know each other)

Nash equilibrium

Definition :

A Nash Equilibrium (NE) is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^* given that every other player j adheres to a_j^*

A NE corresponds to a stable "social norm": *if everyone follows it, no person will wish to deviate from this*

Note that the solution proposed in the bar game in the movie a beautiful mind does not correspond to a Nash equilibrium (Anderson and Enger, 2002)

Nash equilibrium

Assume that (a_i',a_{-i}) is the action profile in which every player j **except i** chooses her action a_j as specified by a, whereas player i deviates to a_i'

Definition :

The action profile a^* in a strategic game is a Nash Equilibrium if for every player i and for every action a_i of player i, a^* is at least as good according to player i's preferences as the action profile (a_i,a_{-i}^*) in which player i chooses a_i while every other player j chooses action a_j^*. Equivalently, for every player i,

$u_i(a^*) \geq u_i(a_i,a_{-i}^*)$ for every action a_i of player i
where u_i is the payoff function that represents player i's preferences

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 / 3	7 / 0
Fink	0 / 7	1 / 1

Assume the profile :

(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 / 3	7 / 0
Fink	0 / 7	1 / 1

Assume the profile :

(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Quiet) $u_1 \rightarrow 7$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

1. The prisoner's dilemma

Assume the profile :

	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1

(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Quiet) $u_1 \rightarrow 7$

(Quiet, Fink) $u_2 \rightarrow 7$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

1. The prisoner's dilemma

Assume the profile :

	Quiet	Fink
Quiet	3, 3	0, 7
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(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Quiet) $u_1 \rightarrow 7$

(Quiet, Fink) $u_2 \rightarrow 7$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

Examples

1. The prisoner's dilemma

Assume the profile :

	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1

(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Quiet) $u_1 \rightarrow 7$

(Quiet, Fink) $u_2 \rightarrow 7$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

(Fink, Quiet) $u_2 \rightarrow 0$

Examples

1. The prisoner's dilemma

Assume the profile :

	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1

(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Quiet) $u_1 \rightarrow 7$

(Quiet, Fink) $u_2 \rightarrow 7$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

(Fink, Quiet) $u_2 \rightarrow 0$

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 / 3	7 / 0
Fink	0 / 7	1 / 1

Note that any deviation from this NE results in a worse outcome.
This NE is therefore also a *strict* NE : for every player i, $u_i(a^*) > u_i(a_i, a_{-i}^*)$ for every action a_i of player i

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

(Fink, Quiet) $u_2 \rightarrow 0$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, swerve) $u_1 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, swerve) $u_1 \rightarrow +1$

(swerve, straight) $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \to 0 \quad u_2 \to 0$

(straight, swerve) $u_1 \to +1$

(swerve, straight) $u_2 \to +1$

(straight, straight) $u_1 \to -10 \quad u_2 \to -10$

(swerve, straight) $u_1 \to -1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \to 0 \quad u_2 \to 0$

(straight, swerve) $u_1 \to +1$

(swerve, straight) $u_2 \to +1$

(straight, straight) $u_1 \to -10 \quad u_2 \to -10$

(swerve, straight) $u_1 \to -1$

(straight, swerve) $u_2 \to -1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(straight, swerve) $u_1 \to +1 \; u_2 \to -1$

(swerve, straight) $u_1 \to -1 \quad u_2 \to +1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(straight, swerve) $u_1 \to +1 \; u_2 \to -1$

(swerve, swerve) $u_1 \to 0$

(swerve, straight) $u_1 \to -1 \quad u_2 \to +1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$
(swerve, swerve) $u_1 \rightarrow 0$
(straight, straight) $u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

43-3

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$
(swerve, swerve) $u_1 \rightarrow 0$
(straight, straight) $u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$
(straight, straight) $u_1 \rightarrow -10$

43-4

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$
(swerve, swerve) $u_1 \rightarrow 0$
(straight, straight) $u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$
(straight, straight) $u_1 \rightarrow -10$
(swerve, swerve) $u_2 \rightarrow 0$

43-5

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -1
Straight	-1 +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$
(swerve, swerve) $u_1 \rightarrow 0$
(straight, straight) $u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$
(straight, straight) $u_1 \rightarrow -10$
(swerve, swerve) $u_2 \rightarrow 0$

43-6

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0, 0	-1, +1
Straight	+1, -1	-10, -10

Both NE are also strict NE

… +1 $u_2 \rightarrow$ -1

(swerve, swerve) $u_1 \rightarrow$ 0

(straight, straight) $u_2 \rightarrow$ -10

(swerve, straight) $u_1 \rightarrow$ -1 $u_2 \rightarrow$ +1

(straight, straight) $u_1 \rightarrow$ -10

(swerve, swerve) $u_2 \rightarrow$ 0

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2 / 2	1 / 0
fish	0 / 1	1 / 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

44-4

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2 / 2	1 / 0
fish	0 / 1	1 / 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

(fish, whale) $u_2 \rightarrow 0$

44-5

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2 / 2	1 / 0
fish	0 / 1	1 / 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

(fish, whale) $u_2 \rightarrow 0$

44-6

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2 / 2	1 / 0
fish	0 / 1	1 / 1

Both NE are also a *strict* NE

$u_2 \rightarrow 2$

1

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

(fish, whale) $u_2 \rightarrow 0$

44-7

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \rightarrow 2$ $u_2 \rightarrow 1$

(Strav., Strav.) $u_1 \rightarrow 1$ $u_2 \rightarrow 2$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \rightarrow 2$ $u_2 \rightarrow 1$

(Strav., Bach) $u_1 \rightarrow 0$

(Strav., Strav.) $u_1 \rightarrow 1$ $u_2 \rightarrow 2$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \rightarrow 2$ $u_2 \rightarrow 1$

(Strav., Bach) $u_1 \rightarrow 0$

(Bach, Strav.) $u_2 \rightarrow 0$

(Strav., Strav.) $u_1 \rightarrow 1$ $u_2 \rightarrow 2$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \rightarrow 2$ $u_2 \rightarrow 1$

(Strav., Bach) $u_1 \rightarrow 0$

(Bach, Strav.) $u_2 \rightarrow 0$

(Strav., Strav.) $u_1 \rightarrow 1$ $u_2 \rightarrow 2$

(Bach, Strav.) $u_1 \rightarrow 0$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \to 2$ $u_2 \to 1$
(Strav., Bach) $u_1 \to 0$
(Bach, Strav.) $u_2 \to 0$

(Strav., Strav.) $u_1 \to 1$ $u_2 \to 2$
(Bach, Strav.) $u_1 \to 0$
(Strav., Bach) $u_2 \to 0$

45-5

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \to 2$ $u_2 \to 1$
(Strav., Bach) $u_1 \to 0$
(Bach, Strav.) $u_2 \to 0$

(Strav., Strav.) $u_1 \to 1$ $u_2 \to 2$
(Bach, Strav.) $u_1 \to 0$
(Strav., Bach) $u_2 \to 0$

45-6

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

Both NE are also a *strict* NE

$u_2 \to 1$
0
(Bach, Strav.) $u_2 \to 0$

(Strav., Strav.) $u_1 \to 1$ $u_2 \to 2$
(Bach, Strav.) $u_1 \to 0$
(Strav., Bach) $u_2 \to 0$

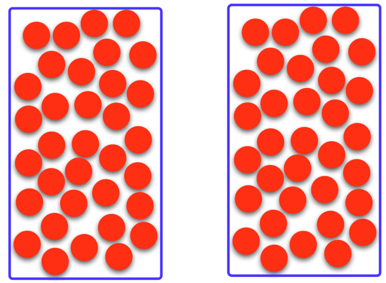
45-7

Population steady state

A NE corresponds to a *steady state* of an interaction between the members of several populations, one for each player in the game

	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1

player 1 player 2



Quiet Fink

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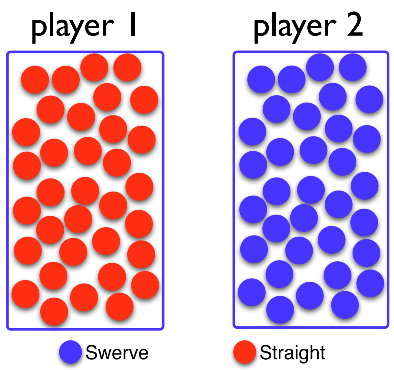
Population steady state

A NE corresponds to a *steady state* of an interaction between the members of several populations, one for each player in the game

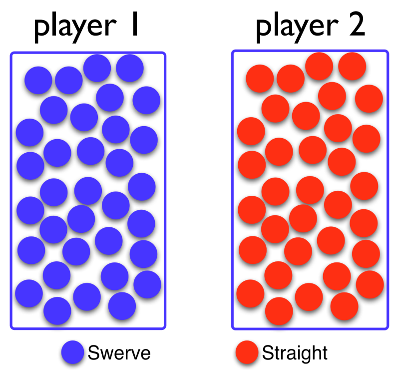
	Swerve	Straight
Swerve	0, 0	-1, +1
Straight	+1, -1	-10, -10



Population steady state

A NE corresponds to a *steady state* of an interaction between the members of several populations, one for each player in the game

	Swerve	Straight
Swerve	0, 0	-1, +1
Straight	+1, -1	-10, -10



Best-response

How to find the Nash Equilibrium in bigger games?

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

For every action a column player chooses there is a subset of best responses of the row player

$B_r(\text{Bach})=\{\text{Bach}\}$ *and* $B_r(\text{Strav})=\{\text{Strav}\}$

best response function of r player

$$B_i(a_{-i})=\{a_i \in A_i : u_i(a_i,a_{-i}) \geq u_i(a_i',a_{-i})\ \forall\ a_i' \in A_i\}$$

Best-response

Best response function can thus be used to define NE

Definition :

The action profile a^* in a strategic game is a Nash Equilibrium if and only if every player's action is a best response to the other player's actions

a_i^* is in $B_i(a_{-i}^*)$ for every player i

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Best-response

Best-response functions can be used to find the NE

METHOD :

STEP 1: find the best-response function for each player

STEP 2: find the action profiles that satisfy : a_i^* is in $B_i(a_{-i}^*)$ for every player i

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Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

Best response of player 2 to player 1

51-1

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Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$

Best response of player 2 to player 1

51-2

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Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$

Best response of player 2 to player 1

51-3

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

$B_2(T)=\{L\}$

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

$B_2(T)=\{L\}$ $B_2(M)=\{L,C\}$

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

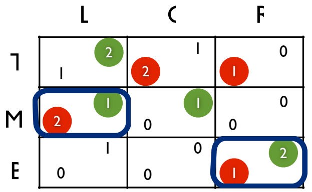
Best response of player 2 to player 1

$B_2(T)=\{L\}$ $B_2(M)=\{L,C\}$ $B_2(B)=\{R\}$

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player



Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

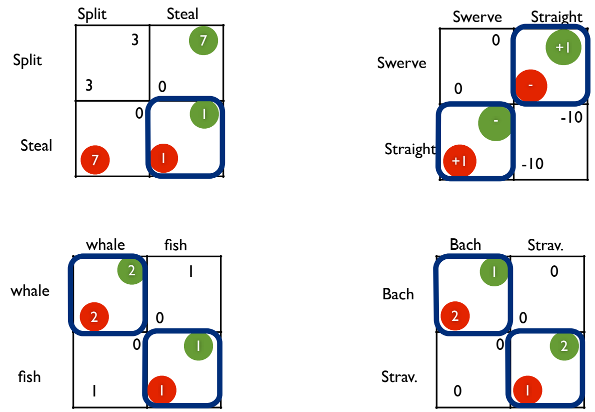
Best response of player 2 to player 1

$B_2(T)=\{L\}$ $B_2(M)=\{L,C\}$ $B_2(B)=\{R\}$

STEP 2: boxes with two coloured payoffs are NE

51-8

Best-response



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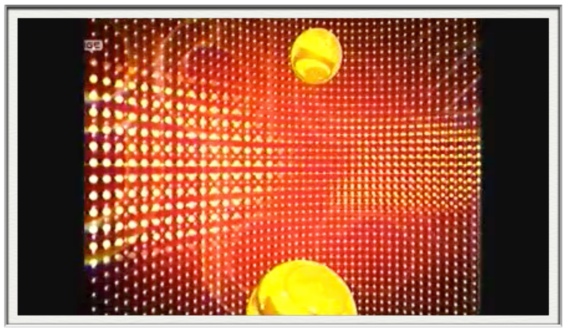
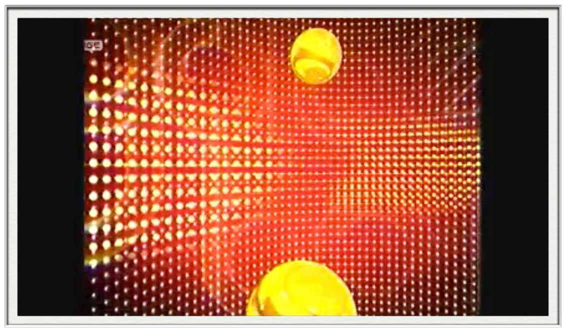
How to beat the game?



Fragment from Golden Balls (ITV1)

53-1

How to beat the game?



Fragment from Golden Balls (ITV1)

53-2

Dominance

In any game, a player's action *strictly dominates* another action if it is superior, no matter what the other player does

	Split	Steal
Split	3 / 3	7 / 0
Steal	0 / 7	1 / 1

Steal strictly dominates Split

If player 2 plays Split, then player 1 prefers Steal

If player 2 plays Steal, then player 1 also prefers Steal

Dominance

Definition :

In a strategic game player *i*'s action a_i'' **strictly** dominates her action a_i' if

$u_i(a_i'',a_{-i}) > u_i(a_i',a_{-i})$ for every list a_{-i} of the other player's action

We say that a_i' is strictly dominated

Strictly dominated actions can never be part of a NE since they are not part of a best response to any actions

Dominance

Definition :

In a strategic game player *i*'s action a_i'' **weakly** dominates her action a_i' if

$u_i(a_i'',a_{-i}) \geq u_i(a_i',a_{-i})$ for every list a_{-i} of the other player's action and

$u_i(a_i'',a_{-i}) > u_i(a_i',a_{-i})$ for **some** list a_{-i} of the other player's action

We say that a_i' is weakly dominated

Dominance

	L	R
T	1	0
M	2	0
B	2	1

No matter what the column player does...

M weakly dominates T

B weakly dominates M

BUT: B strictly dominates T

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Dominance

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

Neither *whale* nor *fish* strictly or weakly dominates the other player's action

	Swerve	Straight
Swerve	0, 0	-, +1
Straight	+1, -	-10, -10

Neither *swerve* nor *straight* strictly or weakly dominates the other player's action

Pareto efficiency

Pareto optimality is a measure of efficiency.

"An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player."

NE and PO

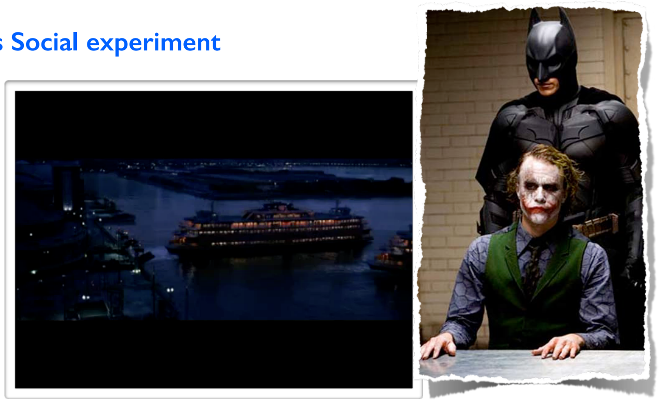
	whale	fish
whale	2, 2 (NE and PO)	0, 1
fish	1, 0	1, 1 (NE)

	Split	Steal
Split	3, 3 (PO)	0, 7
Steal	7, 0	1, 1 (NE)

Shor, Mikhael, "Pareto Optimal," Dictionary of Game Theory Terms, Game Theory .net, <http://www.gametheory.net/dictionary/ParetoOptimal.html> Web accessed: 11/09/2012

Game theory in popular culture

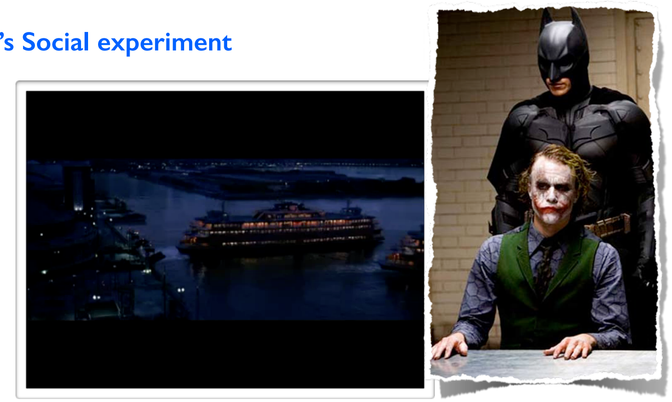
The Joker's Social experiment



What does the payoff matrix look like? Are there any pure Nash equilibria?

Game theory in popular culture

The Joker's Social experiment



What does the payoff matrix look like? Are there any pure Nash equilibria?