Assignment 1: Game theory

General remarks:
_ Mail your results to Luis Martínez <fnxabraxas@gmail.com>.
_ Provide a single (self-contained) *.PDF file.
_ Put your name and your affiliation (VUB/ULB) both on the document and in the file name.

A coordination game

Two people can perform a task if, and only if, they both exert effort. They are both better off if they both exert effort and perform the task than if neither exerts effort (and nothing is accomplished); the worst outcome for each person is that she exerts effort and the other does not (in which case again nothing is accomplished). Specifically, the players' preferences are represented by the expected value of the payoff functions in the figure below, where c is a positive number less than 1 that can be interpreted as the cost of exerting effort.

	No effort	*Effort*
No effort	*0,0*	*0, -c*
Effort	*-c, 0*	*1-c, 1-c*

1. Find all the mixed strategy Nash equilibria of this game. Draw the figures to show your results.
2. How do the equilibria change as c increases? Explain the reasons for the changes.

Two-period Prisoner's Dilemma

Two people simultaneously choose actions; each person chooses either Q or F (as in the Prisoner's Dilemma). Then they simultaneously choose actions again, once again each choosing either Q or F. Each person's preferences are represented by the payoff function that assigns to the terminal history ((W, X), (Y, Z)) (where each component is either Q or F) a payoff equal to the sum of the person's payoffs to (W, X) and to (Y, Z) in the Prisoner's Dilemma given in the following figure:

	Quiet	*Fink*
Quiet	*2,2*	*0,3*
Fink	*3,0*	*1,1*

Specify this situation as an extensive game with perfect information and simultaneous moves and find its subgame perfect equilibria.

Sequential truel

Each of persons A, B, and C has a gun containing a single bullet. Each person, as long as she is alive, may shoot at any surviving person. First A can shoot, then B (if still alive), then C (if still alive).

Denote by p_i the probability that player i hits her intended target; assume that $0 < p_i < 1$. Assume that each player wish to maximize her probability of survival; among outcomes in which her survival probability is the same, she wants the danger posed by any other survivors to be as small as possible.

Model this situation as an extensive game with perfect information and chance moves. (Draw the diagram. Note that the sub-games following histories in which A misses her intended target are the same).
Find the subgame perfect equilibria of the game. (Consider only cases in which p_A, p_B, and p_C are all different.) Explain the logic behind A's equilibrium action.
Show that "weakness is strength" for C: she is better off if $p_C < p_B$ than if $p_C > p_B$.

Success!